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ABSTRACT

This is one of eighteen sets of individualized mathematics problems developed by the Oregon Vo-Tech Math Project. Each of these problem packages is organized around a mathematical topic and contains problems related to diverse vocations. Solutions are provided for all problems. Problems in this volume require solution of linear equations, systems of linear equations, and quadratic equations. Problems are drawn from fifteen vocational areas: agriculture, industrial, electrical and hydraulics technology, forestry, drafting, clerical work, aviation mechanics, diesel mechanics, electronics, marketing, industrial mechanics, fire and police science, wastewater technology, food processing, forest products, and auto mechanics. (SD)

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INDIVIDUALIZED LEARNING SYSTEMS

Individualized Math Problems in Simple Equations

Oregon Vo-Tech Mathematics Problem Sets

Retrieval - Dissemination Center
Oregon Board of Education
942 Lancaster Drive N.E.
Salem, Oregon 97310
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U.S. DEPARTMENT OF HEALTH,
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Individualized Learning Systems

MATHEMATICS

Simple Equations
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SIMPLE EQUATIONS
Level I

Agriculture

A. Problems with solutions.

- 1 We need 100 lbs. of 14% dig. protein feed. We have 100 lbs. of 7% dig. protein feed and 100 lbs. of 17% dig. protein feed. How many lbs. of each do we need to use to obtain the 14% protein feed?

Solution:

Let x = no. of lbs. of 7% feed

100 lbs - x = no. of lbs of 17% feed

$$7\% (x) + 17\% (100 \text{ lbs} - x) = 14\% (100 \text{ lbs})$$

$$.07 x + .17 (100 \text{ lbs} - x) = 14\% (100 \text{ lbs.})$$

$$.07 x + 17 \text{ lbs.} - .17x = 14 \text{ lbs.}$$

$$- .10 x = 3 \text{ lbs.}$$

$$x = \frac{-3 \text{ lbs}}{-.10 \text{ lbs}} = 30 \text{ lbs.}$$

$$(100 \text{ lbs} - x) = (100 \text{ lbs} - 30 \text{ lbs}) = 70 \text{ lbs}$$

Thus 30 lbs. of 7% dig. protein and 70 lbs. of 17% dig. protein.

2. Determine the three largest whole numbers proportional to 2,3,1 representing, respectively, the amounts of nitrogen, phosphoric acid, and potash obtained for a fertilizer formed from cyanamid testing 32% nitrogen, superphosphate testing 35% phosphoric acid, and muriate of potash testing 40% potash, for one ton fertilizer.

Solution:

Consider data for one ton or 2,000 lbs. of fertilizer.

Let x = the number of pounds of potash in 2,000 lbs. of fertilizer.

Then $\frac{x}{0.40}$ = the no. of lbs of muriate of potash to furnish the x lbs. potash.

Let $2x$ = the number of pounds of nitrogen in 2,000 lbs of fertilizer.

Then $\frac{2x}{0.32}$ = the no. of pounds of cyanamid required to furnish $2x$ lbs. of nitrogen.

Let $3x$ = the number of pounds of superphosphate required to furnish $3x$ lbs. of phosphoric acid.

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SIMPLE EQUATIONS
Level I

Agriculture

A. Problems with solutions, continued.

2. Continued.

$$\text{Thus } \frac{2x}{0.32} + \frac{3x}{0.35} + \frac{x}{0.40} = 2,000 \text{ lbs.}$$

$$(1120) \left(\frac{200x}{32} + \frac{300x}{35} + \frac{100x}{40} \right) = (2,000 \text{ lbs.}) (1120)$$

$$7,000x + 9,600x + 2,800x = 2,240,000 \text{ lbs.}$$

$$19,400x = 2,240,000 \text{ lbs}$$

$$x = 115.46 \text{ lbs approx.}$$

x must be a whole number therefore 115 lbs.

Then 230 lbs Nitrogen, 345 lbs. phosphoric acid and 115 lbs potash.

SIMPLE EQUATIONS
Level I

Agriculture

B. Problems without solutions.

3. Mr. Smith has two feeds, one feed contains 5% dig. protein the other contains 19% dig. protein. How much of each should Mr. Smith use to have 100 lbs. of feed mixture containing 13% dig. protein?
4. How many pounds of cream testing 42% butterfat must be added to 2,000 lbs. of 3.5% butterfat milk to make a 12% butterfat cream?
5. If corn is worth \$3.94 per bushel and hogs worth 38¢ per pound of live weight, how many bushel of corn are equal in value to 100 lbs. of live-weight hogs.

SIMPLE EQUATIONS
Level I

Agriculture

B. Problems without solutions, continued.

6. How many pounds of air dried alfalfa testing 64% moisture must be delivered to a mechanical drier to obtain a ton of hay testing 9% moisture?
7. How many pounds of 35% super phosphate should be used with a ton of 4-8-4 fertilizer to raise the phosphoric acid content to 18%?
8. Melba has two feeds. One feed contains 8% dig. protein while the other contains 23% dig. protein. How much of each feed should she use to have 100 lbs. of feed mixture containing 8.5% dig. protein?

SIMPLE EQUATIONS
Level II

Agriculture

B. Problems without solutions, continued.

9. A tractor which has a 24 quart radiator is now filled with 15% anti-freeze, how many quarts of pure antifreeze must be added to make a mixture of 50% antifreeze?
10. Determine the three largest whole numbers proportional to 4,5,2, representing, respectively the percents of nitrogen, phosphoric acid, and potash, obtained for a fertilizer formed from cyanamid testing 26% nitrogen, super phosphate testing 30% phosphoric acid and muriate of potash testing 24% potash.

SIMPLE EQUATIONS
Level II

Agriculture

Complete solutions to B problems.

3. Let x = no. of lbs. of 5% dig. protein.

100 lbs - x = no. of lbs. of 19% dig. protein

$$5\% (x) + 19\% (100 \text{ lbs} - x) = 13\% (100 \text{ lbs})$$

$$.05 x + .19 (100 \text{ lbs} - x) = .13 (100 \text{ lbs})$$

$$.05 x + 19 \text{ lbs} - .19 x = 13 \text{ lbs}$$

$$- .14 x = -6 \text{ lbs}$$

$$x = \frac{-6 \text{ lbs}}{-.14} = 42.9 \text{ lbs.}$$

$$(100 \text{ lbs} - x) = 100 \text{ lbs} - 42.9 \text{ lbs} = 57.1 \text{ lbs.}$$

Thus 42.9 lbs. of 5% dig. protein and 57.1 lbs of 19% dig. protein.

4. Let x = no. of lbs of 42% butterfat

$$42\% (x) + 3.5\% (2,000 \text{ lbs}) = 12\% (2,000 \text{ lbs} + x)$$

$$.42 x + .035 (2,000 \text{ lbs}) = .12 (2,000 \text{ lbs} + x)$$

$$.42 x + 70 \text{ lbs} = 240 \text{ lbs} + .12 x$$

$$.30 x = 170 \text{ lbs}$$

$$x = \frac{170 \text{ lbs}}{.30} = 567 \text{ lbs. of 42\% butterfat cream}$$

5. Let x = no. of bushels of corn

$$\left(\frac{\$3.94}{\text{bushel}} \right) (x) = \left(\frac{\$.38}{\text{pound}} \right) (100 \text{ lbs.})$$

$$\frac{\$3.94 x}{\text{bushel}} = \$38.00$$

$$x = \frac{\$38.00}{\$3.94/\text{bushel}} = 9.6 \text{ bushels of corn}$$

6. Let x = no. of lbs. of 64% moisture hay

$$100\% - 64\% = 36\% \text{ dry matter}$$

$$100\% - 9\% = 91\% \text{ dry matter}$$

$$36\% (x) = 91\% (2,000 \text{ lbs.})$$

$$.36 x = .91 (2,000 \text{ lbs.})$$

$$.36 x = 1,820 \text{ lbs}$$

$$x = \frac{1,820 \text{ lbs}}{.36} = 5,056 \text{ lbs.}$$

SIMPLE EQUATIONS
Level II

Agriculture

Complete solutions to B problems, continued.

7. Let x = no. of pounds of 35% superphosphate

$$8\% (2,000 \text{ lbs}) + 35\% (x) = 18\% (2,000 \text{ lbs} + x)$$

$$.08 (2,000 \text{ lbs}) + .35 x = .18 (2,000 \text{ lbs} + x)$$

$$160 \text{ lbs} + .35x = 360 \text{ lbs} + .18 x$$

$$.17 x = 200 \text{ lbs}$$

$$x = \frac{200 \text{ lbs}}{.17} = 1176 \text{ lbs.}$$

8. Let x = no. of lbs. of 8% dig. protein feed

Let $(100 \text{ lbs} - x)$ = no. of lbs. of 23% dig. protein feed

$$8\% (x) + 23\% (100 \text{ lbs} - x) = 8.5\% (100 \text{ lbs})$$

$$(.08)(x) + (.23)(100 \text{ lbs} - x) = (.085)(100 \text{ lbs})$$

$$.08 x + 23 \text{ lbs} - .23 x = 8.5 \text{ lbs}$$

$$- .15 x = - 14.5 \text{ lbs}$$

$$x = \frac{-14.5 \text{ lbs}}{-.15} = 96.7 \text{ lbs.}$$

$$(100 \text{ lbs} - x) = (100 \text{ lbs} - 96.7) = 3.3 \text{ lbs.}$$

Thus 96.7 lbs. of 8% dig. protein and 3.3% of 23% dig. protein.

9. Let x = no. of quarts that must be removed, and the no. of quarts of pure antifreeze to be added.

$$15\% (24 \text{ qt.}) - 15\% (x) + 100\% (x) = 50\% (24 \text{ qt.})$$

$$.15 (24 \text{ qt.}) - .15 (x) + 1.00 (x) = .50 (24 \text{ qt.})$$

$$36. \text{ qt.} = .15 x + x = 12 \text{ qt.}$$

$$.85 x = 8.4 \text{ qt.}$$

$$x = 9.88 \text{ qt. approx.}$$

10. Consider data for 100 lbs of fertilizer:

Let $4x$ = the number of pounds of nitrogen in 100 lbs. of fertilizer.

Then $\frac{4x}{0.26}$ = the number of pounds of cyanamid required to furnish
4x lbs. of nitrogen.

Let $5x$ = the no. of pounds of phosphoric acid in 100 lbs. of
fertilizer.

Then $\frac{5x}{0.30}$ = the no. of pounds of superphosphate required to furnish
5x lbs. of phosphoric acid.

Let $2x$ = the no. of pounds of muriate of potash to furnish the 100 lbs.
of fertilizer.

SIMPLE EQUATIONS
Level II

Agriculture

Complete solutions to B problems, continued.

10. continued.

Then $\frac{2x}{0.24}$ = the no. of lbs. of muriate of potash to furnish the
2x lbs. of potash.

$$\text{Thus : } \frac{4x}{0.26} + \frac{5x}{0.30} + \frac{2x}{0.24} = 100 \text{ lbs.}$$

$$(1560) \left(\frac{400x}{26} + \frac{500x}{30} + \frac{200x}{24} \right) = (100 \text{ lbs.}) (1,560)$$

$$24,000x + 26,000x + 13,000x = 156,000 \text{ lbs.}$$

$$63,000x = 156,000 \text{ lbs.}$$

$$x = 2.48 \text{ lbs. approx.}$$

x must be a whole no., so $x = 2$

Thus 8 lbs. nitrogen, 10 lbs. phosphoric acid and 4 lbs. potash or
8-10-4

SIMPLE EQUATIONS
Level II

Industrial, Electrical and
Hydraulics

A. Problems with Solutions

1. An electric utility company uses the following formula in connection with a Megger test for Conductor insulation resistance testing.

$$\frac{\text{Rated voltage}}{\text{Capacity in KVA} + 1000} = \text{megohms}$$

For a machine rated 1000 KVA, 2300 volts, the minimum insulation resistance should be how much?

Solution: By direct substitution in the above formula,

$$\frac{2300}{1000 + 1000} = 1.15 \text{ megohms}$$

2. Find the equivalent resistance of a dc circuit which has three parallel resistors of 3, 3 and 5 ohms in series with a group of two parallel resistors of 2 and 6 ohms when the two parallel groups are connected by a 1.5 ohm resistor. Answer to the nearest ohm.

For Resistors in Parallel, $\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$, so $R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$

For 2 Resistors in Parallel, $R = \frac{(R_1)(R_2)}{R_1 + R_2}$

For any number of Resistors in series, $R_t = R_1 + R_2 + R_3 + \dots$

Solution: For the three Resistor group

$$\frac{1}{R_t} = \frac{1}{3} + \frac{1}{3} + \frac{1}{5} \text{ mho}$$

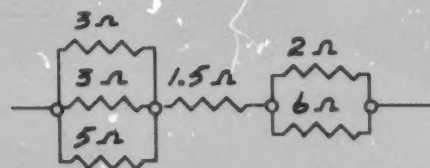
$$= \frac{5+5+3}{15} \text{ mho}$$

$$= \frac{13}{15} \text{ mho} = \frac{15}{13} \text{ ohm}$$

For the two Resistor group

$$R_t = \frac{(2)(6)}{2+6} \text{ ohm}$$

$$= \frac{12}{8} = \frac{3}{2} \text{ ohm}$$



A. Problems with Solutions continued:

$$\text{Total Resistance} = (R_{t_1} + R_{t_2} + 1.5) \text{ ohms}$$

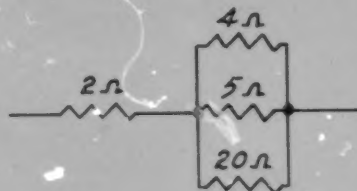
$$= \frac{15}{13} \text{ ohms} + \frac{3}{2} \text{ ohms} + \frac{3}{2} \text{ ohms}$$

$$= 4 \frac{2}{13} \text{ ohms}$$

$$\approx 4 \text{ ohms}$$

B. Problems without solutions

3. What is the equivalent resistance of a dc circuit having a 2 ohm resistance in series with a group of three parallel resistors or 4, 5 and 20 ohms respectively?



4. What voltage is required for a 4 ampere current flow in a 50 ohm resistance dc circuit? $I = \frac{E}{R}$

SIMPLE EQUATIONS
Level II

Industrial, Electrical and
Hydraulics

B. Problems without solutions

5. With only an ammeter and some resistors of known value available, an electrician wishes to check the voltage of a dc source. He connects three resistors, 10, 15, and 30 ohms respectively, in series with the source and measures a current of 2 amperes.

$$I = \frac{E}{R} = \frac{E}{R_1 + R_2 + R_3}$$

What is the source potential?

6. What resistance in a dc circuit with 90 volt potential would result in a 5 ampere current?

$$I = \frac{E}{R}$$

7. In a dc circuit, a voltmeter across the terminals reads 32 volts and the circuit resistance is 5 ohms.

$$I = \frac{E}{R}$$

What is the circuit current?

SIMPLE EQUATIONS
Level II

Industrial, Electrical and
Hydraulics

B. Problems without solutions

8. In a dc series circuit having resistances $R_1 = 5$ ohms, $R_2 = 10$ ohms and $R = 15$ ohms, what battery potential is required to force a 0.5 ampere current through the circuit? Check your solution by calculating the individual IR drops across each resistor.

$$I = \frac{E}{R}$$

9. The regulator rating for a single phase regulator in a single phase circuit is given by the following formula.

$$\text{Regulator KVA} = \frac{\%R \times E \times I}{100}, \text{ where } \%R = \begin{array}{l} \text{maximum percent} \\ \text{regulation range} \\ \text{of the regulator} \end{array}$$

$E =$ single phase voltage in KV
 $I =$ line current in amperes

What would the regulator rating be for a 7200/12,500 volt primary feeder carrying a 174 ampere current load to be regulated over a $\pm 10\%$ range?

B. Problems without solutions

10. Section 4.02.02-Down Guys, From an electrical utility company's District standards reads as follows:

" A down guy may be defined as a tension brace direct from near the unloaded position on the pole to an anchor in the earth, usually at an elevation approximately equal to the elevation of the butt of the pole."

" The lead of the distance out from the base of the pole to the anchor attachment shall normally not be less than $\frac{1}{3}$ nor more than $1\frac{1}{4}$ times the height of the attachment of the down guy above the base of the pole on level ground."

You are to prepare a table which lists the minimum and maximum distances out from the pole base to the anchor attachment for each of the following pole lengths (above ground). Loaded position on the pole is to be figured as $\frac{34}{40}$ of the pole length above ground. Round off lead distances to the nearest foot.

Pole Length	Loaded Position on Pole (above ground)	Lead (Distance out to ground anchor) Round off to nearest foot.	
		Minimum	Maximum
40 ft.			
30 ft.			
25 ft.			
20 ft.			

SIMPLE EQUATIONS
Level II

Industrial, Electrical and
Hydraulics

Complete solutions to B problem

3. For the parallel network:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{5} + \frac{1}{20} = \frac{5 + 4 + 1}{20} = 0.5 \text{ ohms}$$

$$R = \frac{1}{0.5} \text{ ohms} = 2 \text{ ohms}$$

For the series network:

$$R = R_1 + R_2 = (2 + 2) \text{ ohms} = 4 \text{ ohms}$$

$$4. \quad I = \frac{E}{R}$$

$$4 = \frac{E}{50}$$

$$E = (4) (50) \text{ volts} = 200\text{v}$$

$$5. \quad I = \frac{E}{R} = \frac{E}{R_1 + R_2 + R_3}$$

$$2 = \frac{E}{10 + 15 + 30} \text{ volts}$$

$$2(55) \text{ v} = E = 110\text{v}$$

$$6. \quad I = \frac{E}{R}$$

$$5 = \frac{90}{R}$$

$$R = \frac{90}{5} \text{ ohms} = 18 \text{ ohms}$$

7. Use Ohms Law, $I = \frac{E}{R}$, where I = current, E = voltage,
 R = resistance

$$I = \frac{32\text{v}}{5 \text{ ohm}} = 6.4 \text{ amperes}$$

SIMPLE EQUATIONS
Level II

Industrial, Electrical and
Hydraulics

Complete solutions to B problems

$$8. \quad I = \frac{E}{R} = \frac{E}{R_1 + R_2 + R_3}$$

$$0.5 = \frac{E}{5 + 10 + 15} \text{ volts}$$

$$E = 0.5(5+10+15) \quad v = 15v.$$

Check:

$$E_1 = IR_1 = 0.5(5) \quad v = 2.5v.$$

$$E_2 = IR_2 = 0.5(10) \quad v = 5.0v.$$

$$E_3 = IR_3 = 0.5(15) \quad v = 7.5v.$$

$$E_1 + E_2 + E_3 = (2.5 + 5.0 + 7.5)v = 15v.$$

Solution: By direct substitution;

$$KVA = \frac{10 \times 12.5 \times 174}{100} = 217.5, \text{ Rating should be 218 KVA}$$

Level III

10. Pole Length	Loaded Position in Pole	Lead (Dist. out to ground anchor) Round off to nearest foot.	
		Minimum	Maximum
40 ft.	$\frac{34}{40}(40 \text{ ft}) = 34 \text{ ft.}$	$\frac{1}{3}(34\text{ft}) = 11\text{ft. } 4\text{in.}$ $\approx 11\text{ft.}$	$\frac{5}{4}(34\text{ft}) = 42\text{ft. } 6\text{in.}$ $\approx 43\text{ft.}$
30 ft.	$\frac{34}{40}(30 \text{ ft}) = 25\text{ft. } 6\text{in.}$	$\frac{1}{3}(25.5\text{ft}) = 8\text{ft. } 6\text{in.}$ $\approx 9\text{ft.}$	$\frac{5}{4}(25.5\text{ft}) = 31\text{ft. } 11\text{in.}$ $\approx 32\text{ft.}$
25 ft.	$.85(25\text{ft}) = 21\text{ft. } 3 \text{ in.}$	$\frac{1}{3}(21.25\text{ft}) = 7\text{ft. } 1\text{in.}$ $\approx 7\text{ft.}$	$\frac{5}{4}(21.25\text{ft}) = 26\text{ft. } 7\text{in.}$ $\approx 27\text{ft.}$
20 ft.	$.85(20\text{ft}) = 17\text{ft.}$	$\frac{1}{3}(17\text{ft}) = 5.67\text{ft.}$ $\approx 6\text{ft.}$	$\frac{5}{4}(17\text{ft}) = 21.25\text{ft.}$ $\approx 21 \text{ ft.}$

SIMPLE EQUATIONS
Level III

Forestry

A. Problems with solutions.

1. Since the full basal area of a log cannot be cut into boards without loss of sawkerf, deductions must be made. The per cent (A) of total volume lost as sawkerf is given by the formula $A = \frac{100K}{K + T}$ where T is the thickness of the board (inches) and K is the thickness of the sawkerf (in).

Determine A when:

a) Sawkerf 1/4 in, 1 inch boards.

b) Sawkerf 3/16 in, 1 1/2 inch boards.

Solution:

$$a) A = \frac{100K}{K + T} = \frac{(100) (1/4 \text{ in})}{1/4 \text{ in} + 1 \text{ in}} = \frac{\frac{100}{4} \text{ in}}{\frac{5}{4} \text{ in}} = \left(\frac{100}{4} \right) \left(\frac{4}{5} \right) = \frac{100}{5} = 20\%$$

$$b) A = \frac{100K}{K + T} = \frac{(100) \left(\frac{3}{16} \text{ in} \right)}{3/16 \text{ in} + 3/2 \text{ in}} = \frac{\frac{300}{16} \text{ in}}{\left(\frac{3}{16} + \frac{24}{16} \right) \text{ in}} \\ = \frac{\frac{300}{16} \text{ in}}{\frac{27}{16} \text{ in}} = \left(\frac{300}{16} \right) \left(\frac{16}{27} \right) = \left(\frac{300}{27} \right) = 11.1\%$$

SIMPLE EQUATIONS

Forestry

Level II

A. Problems with Solutions, continued.

2. To obtain an estimate for a required volume to make expenses, the logger uses the following formula.

$$V = \frac{(S)(V) + (F)(V) + Y + L + (Tl)(V)}{P} \quad \text{where}$$

V = required annual logging volume

S = cost of stumpage (per MBF)

P = price for which timber is delivered to mill site.

F = falling and bucking cost (per MBF)

Y = yarder annual finance cost

L = Loader annual finance cost

Tl = Truck annual finance cost. (per MBF of subcontracted trucks).

Solve for V in terms of Y, L, P, S, F, and Tl.

Solution:

$$V = \frac{(S)(V) + (F)(V) + Y + L + (Tl)(V)}{P}$$

Multiply each member by P:

$$(V)(P) = (S)(V) + (F)(V) + Y + L + (Tl)(V)$$

Subtract (S)(V), (F)(V), (Tl)(V), from each member:

$$(V)(P) - (S)(V) - (F)(V) - (Tl)(V) = Y + L$$

Factor a V from left member:

$$V(P - S - F - Tl) = Y + L$$

and finally divide each member by (P - S - F - Tl)

$$V = \frac{Y+L}{P-S-F-Tl}$$

SIMPLE EQUATIONS

Forestry

Level II

B. Problems without solutions.

3. For a logger to determine the volume of timber to be cut in order for him to earn expenses he uses the formula:

$$V = \frac{Y+L}{P - S - F - T_1} \text{ where}$$

V = annual logging volume (MBF)

P = price in delivering timber to mill

S = cost of stumpage (per MBF)

F = Felling and bucking cost (per MBF)

Y = yarder annual finance cost

L = loader annual finance cost

T₁ = truck cost/MBF of subcontracted trucking

A logger expects that the local mills will buy his delivered timber at \$120.00/MBF. He owns a yarder (Y= 136,100) and loader (L = 56,000), subcontracts felling and bucking at \$10/MBF and trucking at \$15/MBF and has purchased stumpage at \$75/MBF. Determine the annual logging volume necessary to make expenses.

4. The merchantable height (N) in 16 foot logs of a tree is given by the formula $N = 0.04448 H + 0.275$ when H is the total height above the stump in feet. Solve the formula for H in terms of N.

SIMPLE EQUATIONS

Forestry

Level III

B. Problems without solutions.

5. In order to arrive at the merchantable volume of a log, deductions need to be considered for slab and edgings, and sawkerf. The following formula is commonly used.

$$V = (1-A) (0.0655) (D-S)^2 L$$

where:

V = Board foot volume of log

D = diameter of log (in)

S = slab and edging deduction (in)

L = log length (ft)

A = per cent deduction for sawkerf(decimal)

Determine the merchantable volume for:

a) D = 24 in, S = 2 in, L = 16ft, A = .20

b) D = 30 in, S = 4 in, L = 32 ft, A = .094

Complete solutions to B problems.

3. Solution:

$$P = \$120/\text{MBF}$$

$$S = \$75/\text{MBF} \quad \text{and} \quad Y = \$136,100$$

$$F = \$10/\text{MBF} \quad L = \$56,000$$

$$T1 = \$15/\text{MBF}$$

$$\frac{Y+L}{P-S-F-T1} = \frac{\$136,100 + \$56,000}{\frac{\$120}{\text{MBF}} - \frac{\$75}{\text{MBF}} - \frac{\$10}{\text{MBF}} - \frac{\$15}{\text{MBF}}} = \frac{\$192,100}{\frac{\$20}{\text{MBF}}} = 9,605 \text{ MBF}$$

4. Solution:

$$N = 0.04448 H + 0.275$$

First subtract 0.275 from each side obtaining:

$$N - 0.275 = 0.04448 H$$

Then divide each member by 0.04448 :

$$\frac{N - 0.275}{0.04448} = H$$

So,

$$H = \frac{N - 0.275}{0.04448} = \frac{N}{0.04448} - \frac{0.275}{0.04448}$$

$$H = 22.482 N - 6.1826$$

5. Solution:

$$a) V = (1 - 0.20) (0.0655) (24\text{in} - 2\text{in})^2 (16\text{ft})$$

$$V = (0.80) (0.0655) (22\text{in})^2 16\text{ft}$$

$$V = (0.80) (0.0655) (22\text{in})^2 (192\text{in})$$

$$V = \frac{4869.43 \text{ cu-in}}{144 \frac{\text{cu-in}}{\text{bd ft}}} = 33.8 \text{ bd ft}$$

$$b) V = (1 - 0.096) (0.0655) (30\text{in} - 4\text{in})^2 (32\text{ft})$$

$$V = (0.906) (0.0655) (26)^2 (32\text{ft})$$

$$V = 106.98 \text{ bd ft}$$

SIMULTANEOUS EQUATIONS
Level II

Forestry

A. Problems with solutions.

1. In reseeded a logged area the forest service uses a mix of Intermediate Wheatgrass (0.41/lb), Tall Oatgrass (0.75/lb) and Smooth Bromegrass (0.26/lb). At the rate of 1 lb/acre of seed the Wheatgrass produces 2.3 seeds/sq ft, Oatgrass 3.0 seeds/sq ft and Bromegrass 2.9 seeds/sq ft. If the final mix requires 34.5 seeds/sq ft; a cost of \$6.24/acre, and a total of 13 lbs of seed/acre, now many lbs/acre of each species should be used?

Solution:

Let x = no. lbs/acre of Intermediate Wheatgrass
 y = no. lbs/acre of Tall Oatgrass
 z = no. lbs/acre of Smooth Bromegrass

$$\text{then, } .41x + .75y + .26z = 6.24 \quad (1)$$

$$x + y + z = 13 \text{ and} \quad (2)$$

$$\underline{2.3x + 3.0y + 2.9z = 34.5} \quad (3)$$

Multiplying each side of equation (1) by 100 and (2) by -26 we have

$$\begin{array}{rcl} 41x + 75y + 26z & = & 624 \\ -26x - 26y - 26z & = & -338 \\ \hline 15x + 49y & = & 286 \end{array} \quad (4)$$

Multiplying each side of equation (2) by 129 and (3) by 10 we have

$$\begin{array}{rcl} -29x - 29y - 29z & = & -377 \\ 23x + 30y + 29z & = & 345 \\ \hline -6x + y & = & -32 \end{array} \quad (5)$$

Multiplying equation (5) by -49 and adding at (term by term to 4)

$$\begin{array}{rcl} +294x - 49y & = & +1568 \\ 15x + 49y & = & 286 \\ \hline 309x & = & 1854 \\ x & = & 6 \text{ lbs} \end{array}$$

SIMULTANEOUS EQUATIONS
Level II

Forestry

A. Problems with solutions.

1. continued.

substituting this value for x into equation (4) we obtain

$$\begin{aligned}(15)(6) + 49y &= 286 \\ 90 + 49y &= 286 \\ 49y &= 286 - 90 \\ 49y &= 196\end{aligned}$$

$$y = \frac{196}{49}$$

$$y = 4 \text{ lbs.}$$

then, substituting this value for y and the value for x into equation (2), we obtain:

$$x + y + z = 13$$

$$6 + 4 + z = 13$$

$$10 + z = 13$$

$$z = 13 - 10$$

$$z = 3 \text{ lbs.}$$

B. Problems without solutions.

2. In reseeding a logged area the forest service recommends a mix of intermediate wheatgrass which costs .41/lb and tall Oatgrass which costs .75/lb. At a rate of 1 lb/acre of seed the wheatgrass produces 2.3 seeds/sq. ft and the oatgrass 3.0 seeds/sq.ft. If the final mix requires 25.8 seeds/sq. ft and a cost of \$5.46/acre how many lbs/acre of each species should be used?

SIMULTANEOUS EQUATIONS
Level II

Forestry

Complete solutions to B problems.

2. Solution:

let x = no. lbs/acre of intermediate wheatgrass.

y = no. lbs/acre of oatgrass.

$$\begin{aligned}\text{then: } .41x + .75y &= 5.46 \quad \text{and} \\ 2.3x + 3.0y &= 25.8\end{aligned}$$

now multiply each member of the first equation by -4 obtaining:

$$\begin{aligned}-1.64x - 3.0y &= -21.84 \\ 2.30x + 3.0y &= 25.80 \\ \hline .66x &= 3.96\end{aligned}$$

$$x = \frac{3.96}{.66} = 6 \text{ lbs/acre}$$

then substituting in the second equation:

$$\begin{aligned}2.3(6) + 3.0y &= 25.80 \\ 13.8 + 3.0y &= 25.80 \\ 3.0y &= 12 \\ y &= 4 \text{ lbs/acre}\end{aligned}$$

SIMPLE EQUATIONS
Level 1

Drafting

A. Problems with Solutions

Pascal's Law, which is the basis for modern hydraulics, states that any force applied to a confined liquid is transmitted equally in all directions throughout the liquid regardless of the shape of the container. Using this and the definition of pressure, $P = F/A$, solve each of the following: (Note: P = pressure, F = Force, A = area)

1. What is the total force on a piston, 2 feet in diameter, when the pressure in the cylinder is 100 lb/sq in?

Solution:

$$\text{Since } P = \frac{F}{A} \quad F = PA$$

$$\begin{aligned} \text{The area of the piston} &= (\pi)(r^2) = (\pi)(12 \text{ in})^2 \\ &= (\pi)(144 \text{ sq in}) \\ &= (3.14)(144 \text{ sq in}) \\ &= 452.16 \text{ sq in} \end{aligned}$$

$$\text{Force} = (100 \text{ lbs/sq in})(452.16 \text{ sq in}) = 45,216 \text{ lbs}$$

2. A hydraulic jack has a small cylinder with an area of 2 sq in and a large cylinder with an area of 100 sq in. What is the force on the piston in the large cylinder when the force on the small cylinder is 20 lb?

Solution:

This problem can be worked as a simple proportion

$$\frac{20 \text{ lbs}}{2 \text{ sq in}} = \frac{N}{100 \text{ sq in}}$$

$$2N \text{ sq in} = (20 \text{ lbs})(100 \text{ sq in})$$

$$N = \frac{(20 \text{ lbs})(100 \text{ sq in})}{2 \text{ sq in}}$$

$$N = 1,000 \text{ lbs}$$

B. Problems without Solutions

3. A hydraulic jack has a small cylinder of one square inch in cross-section and a large cylinder of 100 sq inch in cross-section. How many pounds can you lift with each pound of force on the small cylinder?

B. Problems without Solutions (continued)

4. A slab of concrete floor has settled because of poor compaction of the foundation soil. It is proposed to drill holes in the concrete and pump mud through them to raise the slab by creating hydraulic pressure under the slab. What will be the force, in lbs, under a 10 ft by 10 ft slab caused by a mud pump that exerts a pressure of 10 pounds per square inch?

5. A concrete footing 2 feet square rests on a coil that will safely withstand a pressure of 1000 pounds per square foot. What is the total load (force) that may be safely put on the footing?

6. A vertical cylindrical tank 10 feet in diameter and 10 feet high rests on a concrete foundation. a) If the tank is full of water, what is the pressure in pounds per square foot on the floor? (Water weighs 62.43 lbs/cu ft) b) What would be the pressure in pounds per square foot on the floor if the tank were filled with mercury? (Mercury weighs 845.5 lbs/cu ft)

SIMPLE EQUATIONS
Level 1

Drafting

B. Problems without Solutions (continued)

The vertical distance between two elevations in a liquid is known as the "Head" of the liquid on the lower elevation. The relationship between head and pressure is:

$$p = wh \quad h = \text{head in ft} \quad w = \text{weight of 1 cu ft of liquid}$$

Specific gravity "w" of a body is the ratio of the weight of the body to the weight of an equal volume of water.

Buoyant force is vertical force and is equal to the weight of the displaced liquid.

$$F = Ww \quad W = \text{weight} \quad w = \text{specific gravity}$$

Using this information solve each of the following problems:

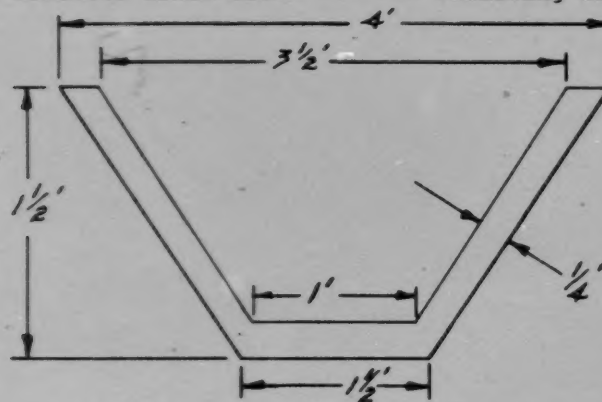
7. What is the pressure head of water that will create a pressure of 65 pounds per square inch?

SIMPLE EQUATIONS
Level 2

Drafting

B. Problems without Solutions (continued)

8. A concrete lined ditch has the following dimensions:



bottom width - 1 foot
depth including freeboard - 1.5 feet
side slopes - 1 to 1
thickness of concrete - 0.25 feet
density of this concrete - 120 lbs/cu ft

At what distance below the top of the concrete lining will a water table in the foundation cause the lining to float when there is no water in the ditch? Note: Area of a trapezoid =

$$\left[\frac{(b_1 + b_2)}{2} \right] h$$

9. A steel pipe line 24 inches in diameter when full of water has a pressure head, at its lower end, of 175 feet. A valve at the upper end is closed, and a valve at the lower end is opened. What is the maximum crushing force per foot of length on the outside of the pipe as the falling water creates a partial vacuum inside?

Drafting

10. An object weighs 65 pounds in air and 42 pounds in water. Find its volume and specific gravity.

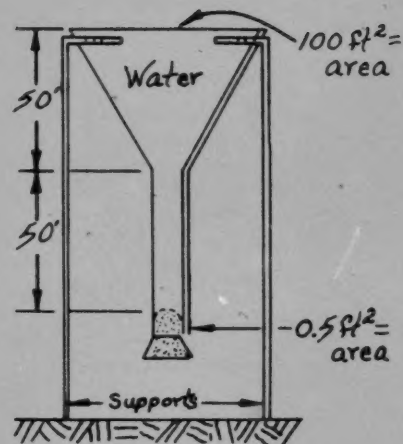
11. A 4 ft diameter sphere floats half submerged in salt water ($w = 64.0$ lbs/cu ft). What minimum weight of concrete ($w = 150.0$ lbs/cu ft) used as an anchor will submerge the sphere completely? Note: Volume of a sphere is $(4/3)(\pi)(r^3)$
12. A plastic pipe $3/4$ inch in diameter and 1320 feet long delivers water 26.4 feet below the pipe intake elevation. A valve is located at the outlet end. What is the pressure in pounds per square inch on the valve when it is closed?

SIMPLE EQUATIONS
Level 2

Drafting

B. Problems without Solutions (continued)

13. What is the force tending to push the plug out of the funnel filled with water as shown in the opposite sketch?



14. Compute the bouyant force acting on an empty, closed pipeline 10 feet long and 2 feet in diameter that is submerged in 4 feet of water.

SIMPLE EQUATIONS

Drafting

Level 1

Complete Solutions to B Problems

$$3. \quad \frac{1 \text{ lbs}}{1 \text{ sq in}} = \frac{X}{100 \text{ sq in}}$$

$$(X)(1 \text{ sq in}) = (1 \text{ lb})(100 \text{ sq in})$$

$$X = \frac{(1 \text{ lb})(100 \text{ sq in})}{1 \text{ sq in}}$$

$$X = 100 \text{ lbs}$$

$$4. \quad \begin{aligned} \text{The area of the floor} &= (\text{length})(\text{width}) \\ &= (10 \text{ ft})(10 \text{ ft}) \\ &= 100 \text{ sq ft} \end{aligned}$$

$$\begin{aligned} \text{Convert to sq in} &= (100 \text{ sq ft})(144 \text{ sq in/sq ft}) \\ &= 14,400 \text{ sq in} \end{aligned}$$

If the mud exerts 10 lbs/sq in, the total force on the slab is $(14,400 \text{ sq in})(10 \text{ lbs/sq in}) = 144,000 \text{ lbs}$.

$$5. \quad \begin{aligned} \text{Since the footing is 2 feet square, it's area is} \\ (2 \text{ ft})(2 \text{ ft}) &= 4 \text{ sq ft.} \end{aligned}$$

To find the total force that can be applied, multiply the area times the maximum force per area.

$$\text{Force} = (4 \text{ sq ft})(1,000 \text{ lbs/sq ft}) = 4,000 \text{ lbs}$$

$$6. \quad \text{a) Pressure} = \frac{\text{Weight}}{\text{Area}}$$

$$\begin{aligned} \text{The area of the floor} &= (\pi)(r^2) \\ &= (3.14)(5 \text{ ft})^2 \\ &= (3.14)(25 \text{ sq ft}) \\ &= 78.5 \text{ sq ft} \end{aligned}$$

$$\text{Weight} = (\text{Volume})(\text{Weight/Volume})$$

$$\begin{aligned} \text{Volume of water} &= (\pi)(r^2)(h) \\ &= (3.14)(5 \text{ ft})^2(10 \text{ ft}) \\ &= 785 \text{ cu ft} \end{aligned}$$

$$\begin{aligned} \text{Weight} &= (785 \text{ cu ft})(62.43 \text{ lbs/cu ft}) \\ &= 49,000 \text{ lbs} \end{aligned}$$

$$\text{Pressure} = \frac{49,000 \text{ lbs}}{78.5 \text{ sq ft}} = 624.2 \text{ lbs/sq ft}$$

Level 1

Complete Solutions to B Problems (continued)

Drafting

6. b) Weight of mercury = (845.5 lbs/cu ft) (785 cu ft)
= 664,000 lbs

$$\text{Pressure} = \frac{664,000 \text{ lbs}}{78.5 \text{ sq ft}} = 8,458.6 \text{ lbs/sq ft}$$

7. Pressure = (Head) (Weight)

$$65 \text{ lbs/sq in} = (\text{Head}) [(62.43 \text{ lbs/cu ft}) (1 \text{ cu ft}/1728 \text{ cu in})]$$

$$65 \text{ lbs/sq in} = (\text{Head}) (0.036 \text{ lbs/cu in})$$

$$\text{Head} = \frac{65 \text{ lbs/sq in}}{0.036 \text{ lbs/cu in}} = 1800 \text{ in} = 150 \text{ ft}$$

8. To work this, first find the cross-sectional area of the ditch wall. The area of the ditch wall equals the area of the large trapezoid minus the area of the small trapezoid.

$$\begin{aligned} \text{Area of wall} &= \left[\frac{(4' + 1 \frac{1}{2}')}{2} \right] [1 \frac{1}{2}'] - \left[\frac{(3 \frac{1}{2}' + 1')}{2} \right] [1 \frac{1}{4}'] \\ &= 4.13 \text{ sq ft} - 2.81 \text{ sq ft} \\ &= 1.32 \text{ sq ft} \end{aligned}$$

Therefore, a one foot length of this ditch has a volume of $(1.32 \text{ sq ft})(1 \text{ ft}) = 1.32 \text{ cu ft}$ in its walls. The weight of concrete in a one foot length = $(1.32 \text{ cu ft})(120 \text{ lbs/cu ft}) = 158.4 \text{ lbs}$.

So, you must find how much displaced water would weigh 158.4 lbs and how high on the ditch would this water be. First, how much water weighs 158.4 lbs?

$$\frac{158.4 \text{ lbs}}{62.43 \text{ lbs/cu ft}} = 2.54 \text{ cu ft}$$

Now find how high on the ditch walls this is. In a one foot length of ditch there would need to be 2.54 sq ft of cross-section to displace enough water to float the ditch.

The cross-sectional area of the ditch is

$$\left[\frac{(N' + 1')}{2} \right] \left[\frac{(N' - 1')}{2} \right] \quad \text{where } N \text{ is the top of the trapezoid.}$$

SIMPLE EQUATIONS

Drafting

Level 2

Complete Solutions to B Problems (continued)

8. (continued)

If it equals 2.54 sq ft, then

$$\frac{N^2 - 2N - 1}{4} = 2.54 \text{ sq ft}$$

$$N = \frac{2 \pm \sqrt{4 + 44.64}}{2}$$

$$N = 4.49 \text{ ft}$$

$$\text{The height} = \frac{4.49 \text{ ft} - 1 \text{ ft}}{2} = \frac{3.49 \text{ ft}}{2} = 1.75 \text{ ft}$$

so the ditch won't float.

9. Since the pipe has a pressure head of 175 ft, it is 175 ft long. Therefore, the volume of water in this pipe is $\pi r h$,

$$\begin{aligned} \text{Volume} &= (\pi) (1')^2 (175') \\ &= (3.14) (175 \text{ cu ft}) \\ &= 549.5 \text{ cu ft} \end{aligned}$$

$$\begin{aligned} \text{Weight of water} &= (549.5 \text{ cu ft}) (62.43 \text{ lbs/cu ft}) \\ &= 34,305 \text{ lbs} \end{aligned}$$

$$\begin{aligned} \text{Area of outside of pipe} &= (\text{Circumference}) (\text{Length}) \\ &= (\pi) (D) (175') \\ &= (\pi) (2') (175') \\ &= (3.14) (350 \text{ sq ft}) \\ &= 1,099 \text{ sq ft} \end{aligned}$$

$$\text{Pressure on inside of pipe} = \frac{34,305 \text{ lbs}}{1,099 \text{ sq ft}} = 31.22 \text{ lbs/sq ft}$$

$$\text{Convert to lbs/sq in} = (31.22 \text{ lbs/sq ft}) \left(\frac{1 \text{ sq ft}}{144 \text{ sq in}} \right)$$

$$= 0.22 \text{ lbs/sq in}$$

Difference in pressure inside & outside:

$$14.7 \text{ lbs/sq in} - 0.22 \text{ lbs/sq in} = 14.48 \text{ lbs/sq in}$$

In one foot of pipe there are $(3.14) (24") (12") = 904.32 \text{ sq in}$
So each foot of pipe would undergo a crushing force of

$$(14.48 \text{ lbs/sq in}) (904.32 \text{ sq in}) = 13,095 \text{ lbs}$$

SIMPLE EQUATIONS

Drafting

Level 2

Complete Solutions to B Problems (continued)

10. The difference in the objects weight in water and in air is the bouvancy supplied by the displaced water.

$$\text{Weight of displaced water} = 65 \text{ lbs} - 42 \text{ lbs} = 23 \text{ lbs}$$

$$\text{Volume} = \frac{23 \text{ lbs}}{62.43 \text{ lbs/cu ft}} = 0.37 \text{ cu ft}$$

$$\text{Density} = \frac{65 \text{ lbs}}{0.37 \text{ cu ft}} = 176.43 \text{ lbs/cu ft}$$

$$\text{Specific gravity} = \frac{176.43 \text{ lbs/cu ft}}{62.43 \text{ lbs/cu ft}} = 2.83$$

11. Since the weight of the entire sphere is needed to submerge half of the sphere, it is necessary to find the weight of concrete required to submerge the hemisphere left out of water.

First calculate the volume still to be submerged.

$$\begin{aligned} \text{Volume of the sphere} &= (4/3) (\pi) (2')^3 \\ &= 33.51 \text{ cu ft} \end{aligned}$$

$$\begin{aligned} \text{Weight of displaced water} &= (33.51 \text{ cu ft}) (62.43 \text{ lbs/cu ft}) \\ &= 2,092 \text{ lbs} \end{aligned}$$

So you need

$$\frac{2092 \text{ lbs}}{150 \text{ lbs/cu ft}} = 13.95 \text{ cu ft of concrete to offset}$$

the bouvancy produced by the displaced water.

12. Pressure = (Head) (weight of 1 cu ft of water)
 = (26.4 ft) (62.43 lbs/cu ft)
 = 1,648.15 lbs/sq ft

convert to lbs/sq in

$$\frac{1,648.15 \text{ lbs/sq ft}}{144 \text{ sq in/sq ft}} = 11.45 \text{ lbs/sq in}$$

13. Pressure = (Head) (weight of 1 cu ft of water)
 = (10 ft) (62.43 lbs/cu ft)
 = 624.3 lbs/sq ft

The force on the plug is:

$$(624.3 \text{ lbs/sq ft}) (0.5 \text{ sq ft}) = 312.15 \text{ lbs}$$

SIMPLE EQUATIONS

Drafting

Level 2

Complete Solutions to B Problems (continued)

14. Bouyancy equals the weight of the displaced liquid. So, bouyancy equals the volume of the pipe times the weight per volume of water.

$$\begin{aligned}\text{Volume} &= (\pi) (r^2) (h) \\ &= (3.14) (1')^2 (10') \\ &= (3.14) (10 \text{ cu ft}) \\ &= 31.4 \text{ cu ft}\end{aligned}$$

$$\begin{aligned}\text{Bouyancy} &= (31.4 \text{ cu ft}) (62.43 \text{ lbs/cu ft}) \\ &= 1,960 \text{ lbs}\end{aligned}$$

SIMPLE EQUATIONS
Level 1

Clerical

A. Problems with Solutions

1. How much will each of three people receive if \$180 is to be divided so that the second receives $\frac{2}{3}$ as much as the first and the third receives $\frac{1}{2}$ as much as the second?

Solution: $X + \frac{2}{3}X + \frac{1}{2}(\frac{2}{3}X) = 180$
 $2X = 180$
 $X = 90$

First person receives \$90
Second person receives \$60
Third person receives \$30

2. A suit cost \$65. What must the suit be sold for if a profit of 20% of the selling price is to be made?

Solution: Profit = 0.20 X selling price
Selling Price = cost + profit

$$\begin{aligned} SP - \$65 &= (0.20)(SP) \\ SP - 0.20SP &= \$65 \\ 0.8SP &= \$65 \\ SP &= \$81.25 \end{aligned}$$

3. Mr. Goodman invests part of \$7,500 at 3% annual interest and the rest at 5% annual interest. The income from the two investments is \$279 annually. How much did Mr. Goodman invest at each rate?

Solution:

List the quantities you will use:

Amount invested at 3% = X dollars
Amount invested at 5% = 7,500 - X dollars
Income from the 3% investment = 0.03X dollars
Income from the 5% investment = 0.05 (7500 - X) dollars
Income from both investments = 279 dollars

$$\begin{aligned} 0.03X + 0.05(7,500 - X) &= \$279 \\ 0.03X + \$375 - 0.05X &= \$279 \\ \$96 &= 0.02X \\ \$4,800 &= X \end{aligned}$$

\$4,800 invested at 3%
\$2,700 invested at 5%

SIMPLE EQUATIONS
Level 1

Clerical

B. Problems without Solutions

4. A grocer mixed nuts selling at 40¢ a pound with some selling for 60¢ a pound. If he had 200 lb which he could sell for 48¢ a pound, how many pounds of the 60¢ nuts did he use?

Solution:

5. A discount store owner pays \$15 for a transistor radio on which he wishes to make a 30% profit based on the selling price after he has allowed a discount of 10% to his customer. What should he set as the list price of the radio?

Solution:

6. Two men were partners. The agreement was that one man's profit was to be 25% greater than that of the other man. The profit to be divided one year was \$52,000. How much did each partner receive?

Solution:

SIMPLE EQUATIONS
Level 1

Clerical

B. Problems without Solutions (continued)

7. One card sorter can process a deck of punched cards in 30 minutes, while another can sort the deck in 45 minutes. How long, would it take the two sorters together to process the cards?

Solution:

8. In a factory, one worker can complete a job with a punch press in 7 days, but it takes another worker 8 days to complete the same job with the same kind of machine. If both machines are operated at once, how long will it take to do this job?

Solution:

9. When an article costs \$6, for at least what price should it be sold if the profit is to be at least 20% of the selling price?

Solution:

10. An automobile costs a dealer \$1,300. What must his selling price be to give him 20% of the selling price as overhead and 15% of the selling price for profit?

Solution:

SIMPLE EQUATIONS
Level 1

Clerical

Complete Solutions to B Problems

4.
$$(\$0.60/\text{lb})X + (\$0.40/\text{lb})(200 \text{ lb} - X) = (200 \text{ lb})(\$0.48/\text{lb})$$
$$(\$0.60/\text{lb})X + \$80 - (\$0.40/\text{lb})X = \$96$$
$$(\$0.20/\text{lb})X = \$16$$
$$X = \frac{\$16}{\$0.20/\text{lb}}$$
$$X = 80 \text{ lbs}$$

Therefore, grocer used 80 lbs of nuts selling at \$0.60 per pound

5.
$$\text{Selling Price} = \text{Cost} + \text{Profit} + \text{Discount}$$
$$SP = \$15 + (0.30)SP + (0.10)SP$$
$$(0.60)SP = \$15$$
$$SP = \$25$$

6.
$$X + (1.25)X = \$52,000$$
$$(2.25)X = \$52,000$$
$$X = \$23,111.11$$

Therefore, one partner receives \$23,111.11 of profit and the other partner receives \$28,888.89 of profit

7.
$$X = \text{Time, in minutes, it takes both machines to process the cards}$$
$$1/30 + 1/45 = 1/X$$
$$45 + 30 = 1350/X$$
$$X = \frac{1350}{75}$$
$$X = 18 \text{ min}$$

8.
$$X = \text{Number of days to complete job}$$
$$1/7 + 1/8 = 1/X$$
$$8X + 7X = 56$$
$$15X = 56$$
$$X = 3 \frac{11}{15} \text{ days}$$

9.
$$\text{Selling Price} = \text{Cost} + \text{Profit}$$
$$SP = \$6 + (0.20)SP$$
$$(0.80)SP = \$6$$
$$SP = \$7.50$$

10.
$$\text{Selling Price} = \text{Cost} + \text{Profit} + \text{Overhead}$$
$$\text{Profit} = (0.15)SP$$
$$\text{Overhead} = (0.20)SP$$
$$SP = \$1,300 + (0.15)SP + (0.20)SP$$
$$(0.65)SP = \$1,300$$
$$SP = \$2,000$$

SYSTEMS OF EQUATIONS
Level 2

Clerical

A. Problems with Solutions

1. A grocer blends teas worth 66¢ and 48¢ a pound. If he interchanges the amounts, he saves \$6 in a blend of 100 pounds. Find the ratio of the weight of the two teas in the original blend.

Solution:

x = amount of \$0.66 a pound tea

y = amount of \$0.48 a pound tea

$$x + y = 100$$

$$(\$0.66)x + (\$0.48)y - \$6 = (\$0.48)x + (\$0.66)y$$

$$(\$0.66)(100-y) + (\$0.48)y - \$6 = (\$0.48)(100-y) + (\$0.66)y$$

$$\$66 - (\$0.66)y + (\$0.48)y - \$6 = \$48 - (\$0.48)y + (\$0.66)y$$

$$\$60 - (\$0.18)y = \$48 + (\$0.18)y$$

$$\$12 = (\$0.36)y$$

$$33.33 = y$$

$$x = 100 - 33.33$$

$$x = 66.67$$

$$\text{ratio is } 66.67 : 33.33 = 2 : 1$$

2. Payless makes a profit of 2¢ on each bottle of brand x aspirin that it sells and a profit of 6¢ on each bottle of brand y aspirin that it sells. How many bottles of each brand must the druggist sell if he is to make a total profit of \$8 from the sales of these two brands of aspirin if the total number of bottles sold is 280?

Solution:

x = number of bottles of Brand x sold

y = number of bottles of Brand y sold

$$(\$0.02)x + (\$0.06)y = \$8$$

$$(\$0.02)(280 - y) + (\$0.06)y = \$8$$

$$\$5.60 - (\$0.02)y + (\$0.06)y = \$8$$

$$(\$0.04)y = \$2.40$$

$$y = 60 \text{ bottles}$$

$$x + 60 = 280$$

$$x = 220 \text{ bottles}$$

$$x + y = 280$$

A. Problems with Solutions (continued)

3. A teller mistakenly reversed the two digits in the face amount of a check, overpaying \$9. If the sum of the digits was 9, determine the amount for which the check was drawn.

Solution:

x = tens digit on check

y = ones digit on check

$$10x + y + 9 = 10y + x$$

$$10(9-y) + y + 9 = 10y + (9-y)$$

$$99 - 9y = 9y + 9$$

$$90 = 18y$$

$$5 = y$$

$$x + 5 = 9$$

$$x = 4 \quad \text{amount of check was \$45}$$

B. Problems without Solutions

4. In two years, a sum of money at simple interest amounted to \$330, while in five years it amounted to \$375. What was the sum, and what was the interest rate?

Solution:

SYSTEMS OF EQUATIONS
Level 2

Clerical

B. Problems without Solutions (continued)

5. How fast does each operate if two sorters process 1,200 cards in one minute, or 1,200 cards in two minutes when one breaks down after operating one-half minute?

Solution:

6. A mail-order firm uses the following equation to estimate the number of dollars of sales for the day from the weight of first-class mail received by 7 a.m.

$\text{Sales} = A + B(X)$ where X is the weight of the mail.

If the number of shipping clerks needed is one per \$12,000 of sales, find the number of clerks needed on a day when the mail weights 300 lbs given.

$$\begin{aligned} 11A + 11B &= 49,620 \\ 115A + 1,595B &= 683,700 \end{aligned}$$

Solution:

B. Problems without Solutions (continued)

7. The butcher is to buy some chickens and ducks. He will sell the chickens at a profit of 2¢ per lb and will lose 1¢ per lb on the ducks. He does not wish to make a profit since these items will be used to attract customers. Nor does he intend to lose money on the sales. The suppliers will sell to him at attractive prices only if he buys the same number of pounds of ducks as of chickens. How many pounds of each should the butcher buy in order to meet these conditions.

Solution:

8. The same butcher decides to make a profit of \$20 on the sale of chickens and ducks. The market has changed, and now he must buy 100 lbs more of ducks than of chickens. How many pounds of each should he buy?

Solution:

B. Problems without Solutions (continued)

9. Suppose that the butcher now makes 2¢ per lb on chickens and loses 1¢ per lb in ducks. He wishes to make \$10 for handling costs. The supplier now requires that he buy in such a way that the pounds of chickens purchased exceed by 500 lbs one-half the number of pounds of ducks.
- (a) How many pounds of ducks should he buy?
 - (b) How many pounds of chickens should he buy?
 - (c) If he buys 300 lbs of chickens, can he achieve his goal of \$10?
 - (d) If he buys 150 lbs of ducks, can he achieve his goal?

Solution:

SYSTEMS OF EQUATIONS
Level 2

Clerical

Complete Solutions to B Problems

4. $I = P R T$

$$\$330 = 2RP + P$$

$$\$375 = 5RP + P$$

$$\$330 = P(2R+1)$$

$$\$375 = P(5R+1)$$

$$P = \frac{\$330}{2R+1}$$

$$P = \frac{\$375}{5R+1}$$

$$(\$330)(5R+1) = (\$375)(2R+1)$$

$$\$1,650R + \$330 = \$750R + \$375$$

$$\$900R = \$45$$

$$R = \frac{\$45}{\$900} = 0.05 \quad \text{Interest rate was } 5\%$$

$$P = \frac{\$330}{2(0.05)+1} = \frac{\$330}{1.1} = \$300$$

Principal was \$300

5. x = cards per minute processed by first machine

$$y$$
 = cards per minute processed by second machine

$$x + y = 1,200$$

$$2x + 1/2y = 1,200$$

$$2x + 1/2(1,200-x) = 1,200$$

$$2x + 600 - 1/2x = 1,200$$

$$1\frac{1}{2}x = 600$$

$$x = 400$$

$$400 + y = 1,200$$

$$y = 800$$

First machine processes 400 cards per minute

Second machine processes 800 cards per minute

SYSTEMS OF EQUATIONS
Level 2

Clerical

Complete Solutions to B Problems (continued)

$$\begin{array}{rcl}
 6. & 11A + 115B & = 49,620 \\
 & 115A + 1,595B & = 683,700 \\
 & 17,545A + 183,425B & = 79,143,900 \\
 - & 13,225A + 183,425B & = 78,625,500 \\
 \hline
 & 4,300A + 0B & = 518,400
 \end{array}$$

$$A = \frac{518,400}{4,320} = 120$$

$$\begin{array}{l}
 (11)(120) + 115B = 49,620 \\
 115B = 48,300
 \end{array}$$

$$B = \frac{48,300}{115} = 420$$

$$\begin{array}{l}
 \text{Sales} = A + B(x) \\
 = \$120 + (\$420)(300) \\
 = \$126,120
 \end{array}$$

$$\text{Number of clerks needed} = \frac{\$126,120}{\$12,000/\text{clerk}} = 10.51 \text{ clerks}$$

Therefore 11 clerks are needed

$$7. \quad x = \text{pounds of chicken}$$

$$y = \text{pounds of duck}$$

$$\$0.02x - \$0.01y = 0$$

$$x = y$$

$$\$0.02x - \$0.01x = 0$$

$$\$0.01x = 0$$

Conditions cannot be met

$$8. \quad x = \text{pounds of chicken}$$

$$y = \text{pounds of duck}$$

$$\$0.02x - \$0.01y = \$20$$

$$\$0.02x - (\$0.01)(x+100) = \$20$$

$$\$0.02x - \$0.01x - \$1.00 = \$20$$

$$\$0.01x = \$21$$

$$x = \frac{\$21}{\$0.01} = 2,100$$

$$2,100 + 100 = y$$

$$y = 2,200$$

Therefore he should buy 2,100 lbs of chicken
and 2,200 lbs of duck

SYSTEMS OF EQUATIONS
Level 2

Clerical

Complete Solutions to B Problems (continued)

9. x = pounds of chicken

y = pounds of duck

$$\$0.02x - \$0.01y = \$10$$

$$x - 500 = y/2$$

$$\$0.02x - (\$0.01)(2x - 1,000) = \$10$$

$$2x - 1,000 = y$$

$$\$0.02x - \$0.02x + \$10 = \$10$$

$$\$0.02x = \$0.02x$$

A + B - any number as long as x and y are in the ratio $x = y/2 + 500$

C. $x = 300$

$$(\$0.02)(300) - \$0.01y = \$10$$

$$\$6 - \$0.01y = \$10$$

$$-\$0.01y = \$4$$

$$y = -400$$

The goal of \$10 cannot be made because he cannot purchase a negative poundage of ducks

D. $y = 150$

$$\$0.02x - (\$0.01)(150) = \$10$$

$$\$0.02x - \$1.50 = \$10$$

$$\$0.02x = \$11.50$$

$$x = \frac{\$11.50}{\$0.02} = 575$$

He can achieve his goal of \$10

SIMPLE EQUATIONS
Level 1

Aviation Mechanics

A. Problems with Solutions

1. If a jet engine is developing 10,000 lb thrust and is driving an airplane at 600 mph, what is the thp (thrust horsepower)?

Solution:

1 hp is equal to 33,000 ft-lb per min or 375 mile-lb per hr.
thp is then obtained by

$$\begin{aligned} \text{thp} &= \frac{(\text{thrust (lb)}) (\text{airspeed (mph)})}{375} \\ &= \left[\frac{(10,000) (600)}{375} \right] \approx 16,000 \end{aligned}$$

2. If a current of 5 amp is flowing through a resistor with a value of 3 ohms, what will the voltage drop be across the resistor?

Solution:

$$E = I R$$

$$= (5 \text{ amp}) (3 \text{ ohm}) = 15 \text{ volts}$$

B. Problems without Solutions

3. Approximately, what power output would you expect to obtain from a good aircraft engine consuming 60 lb (10 gal) per hour of gasoline? Figure the heat value of the fuel as 20,000 BTU per lb.

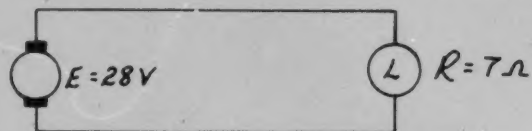
SIMPLE EQUATIONS
Level 1

Aviation Mechanics

B. Problems without Solutions (continued)

4. If 24 volts is applied to a circuit having a resistance of 6 ohms, what will be the current flow? ($E = I R$)

5. Assume that the voltage of the generator is 28, and the resistance of the lamp is 7 ohms. What is the amperage? ($W = I E$)



6. If a current of 20 amp is flowing in a circuit in which the voltage is 110, what is the power being consumed in the circuit? What is this in horsepower? (Work = (current) (force) = $I E$;
1 hp = 746 watts.)

Complete Solutions to B Problems

3. One lb of gasoline is used per minute and has a heat value of 20,000 BTU. One BTU can do 778 ft-lb of work, and since 1 hp $\approx$ 33,000 ft-lb/min, we have

$$\begin{aligned}\text{Power} &= (778 \text{ ft-lb/BTU}) (20,000 \text{ BTU/min}) \left(\frac{1 \text{ HP}}{33,000 \text{ ft-lb/min}} \right) \\ &= 471.52 \text{ hp}\end{aligned}$$

4. By Ohm's Law, $E = I R$

$$\text{or } I = \frac{E}{R} = \frac{24}{6} = 4 \text{ amps}$$

5. It follows that $I = \frac{E}{R} = \frac{28}{7} = 4 \text{ amps}$

6. According to the formula $W = I E$, where W is the power in watts, I is the current in amps, and E is the electromotive force in volts.

$W = (20)(110) = 2200 \text{ watts}$. To obtain horsepower, we divide 2200 by 746 (1 hp $\approx$ 746 watts) to obtain about 2.95 hp

SIMPLE EQUATIONS
Level 1

Diesel Mechanic

A. Problems with Solutions

1. If 4 ampere of current flows through a 10-ohm resistance, what is the applied voltage? (E = volts, I = amperes, and R = ohms.)

Solution:

$$E = IR, E = 4(10) = 40 \text{ volts}$$

2. Solve the formula $E = I R$ for I.

Solution:

In $E = I R$, divide both sides by R and we get $\frac{E}{R} = I$

3. When 10 volts is applied across 5 ohms, how much current will there be?

Solution:

$$I = \frac{E}{R} = \frac{10}{5} = 2 \text{ amperes}$$

4. A series circuit is made up of a 3 ohm resistance, a 2.5 ohm resistance, and a 2.5 ohm resistance. What is the total resistance?

Solution:

In a series circuit, the sum of the individual resistances is the total resistance. $3 + 2.5 + 2.5 = 8 \text{ ohm}$

Level 2

5. A parallel circuit is made up of a 2 ohm resistance, a 3 ohm resistance, and a 6 ohm resistance. What is the total circuit resistance?

Solution:

In a parallel circuit, the total is equal to the reciprocal of the sum of the reciprocals of the individual resistances.

$$\frac{1}{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}} = 1 \text{ ohm}$$

SIMPLE EQUATIONS
Level 3

Diesel Mechanic

A. Problems with Solutions (continued)

6. A piece of lead weighs 22 pounds. The specific gravity of lead is 11.4. A cubic foot of water weighs 1,000 ounces. What is the volume of the lead? (In cu ft and in cu in.)

Solution:

The weight of a material can be found by multiplying the volume by the specific gravity by 62.5 or (1000/16). Thus, if x represents the volume of lead,

$$x(11.4)(62.5) = 22 \text{ lbs}$$

$$x = \frac{22}{(11.4)(62.5)} \text{ cu ft}$$

$$= 0.0309 \text{ cu ft or } (0.0309 \text{ cu ft})(1728 \text{ cu in/cu ft}) = 53.4 \text{ cu in}$$

B. Problems without Solutions
Level 1

7. A coil of wire has a resistance of 4 ohms. In order to cause 4 amps to flow through it, what voltage would be required?
8. A source of 5 volts applied across a resistance of 1,000 ohms produces how much current?
9. A truck's headlight having a resistance of 1.8 ohms is connected to a generator producing 14.5 volts. What current flows through the headlight?

SIMPLE EQUATIONS
Level 1

Diesel Mechanic

B. Problems without Solutions (continued)

10. Solve the formula $E = I R$ for R .

11. If a generator field coil was connected to a 12 volt battery and had a current flow of 1.5 amps, what would be the resistance of the coil?

12. Three 1-ohm resistances are connected in parallel. What is their combined equivalent resistance?

Level 2

13. A 1-ohm resistance is connected in parallel with a 2-ohm resistance. What is their combined equivalent resistance?

Diesel Mechanic

14. A parallel circuit with 20 volts applied across two branches has a total line current of 5 amps. One branch resistance equals 5 ohms. The other branch has what resistance?

15. A string of two 1000-ohm resistances is in series with a parallel bank of two 1000-ohm resistances. What is the total resistance of the series-parallel circuit?

16. With two resistances in parallel of 4 ohms and 3 ohms, what current will 110 volts cause to flow?

17. The resistance of copper of a certain size is 2.521 ohms per 1,000 feet. What is the resistance of 400 feet?

SIMPLE EQUATIONS
Level 1

Diesel Mechanic

Complete Solutions to B Problems

7. $E = I R$ $E = (4)(4) = 16$ volts

8. $I = \frac{5}{1000} = 0.005$ amps

9. $14.5/1.8 \approx 8.06$ amp (nearest hundredth)

10. In $E = I R$, divide both sides by I and we get $\frac{E}{I} = R$

11. $\frac{E}{I} = \frac{12}{1.5} = 8$ ohms

12. $\frac{1}{1/1 + 1/1 + 1/1} = 1$ ohm

Level 2

13. $\frac{1}{1/1 + 1/2} = \frac{1}{3/2} = 2/3$ ohm

14. The current across the 5-ohm resistance is $20/5 = 4$ amp.
This leaves 1 amp through the other resistance. Thus,
 $20/1 = 20$ ohms.

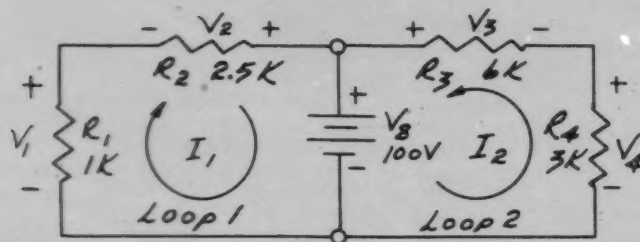
15. The resistance of the parallel part is $\frac{1}{1/1000 + 1/1000} = 500$ ohm.
In the series then, $1000 + 1000 + 500 = 2500$ ohms.

16. The total resistance is: $I = \frac{E}{R} = \frac{110}{1/3 + 1/4} = \frac{110}{7/12} = \frac{1320}{7} = 188.6$ amps

17. Using a proportion, $\frac{x \text{ ohm}}{2.521 \text{ ohm}} = \frac{400 \cancel{\mu\text{C}}}{1000 \cancel{\mu\text{C}}}$ and $x = 1.0084$ ohm

A. Problems with Solutions

1. A 100-Volt battery is feeding two parallel circuit loops as indicated below:



- Using OHM's Law, find I_1 , I_2 , V_1 , V_2 , V_3 , V_4 . Calculate to three decimal digits.
- Prove Kirchoff's voltage law for each of the loops, noting that the voltage drops across the resistors have an opposite sign (polarity) from that of the battery.
- Compute the power in watts dissipated by each resistor.
- Determine the total power delivered by the battery.

- Ref: 1. Kirchoff's Voltage Law (paraphrased): "The algebraic sum of the voltages around any closed circuit loop is equal to zero."
 2. OHM's Law: $V = IR$ V in Volts, I in Amperes, R in OHMs
 3. Watt's Law: $P = VI$ V in Volts, I in Amperes, P in Watts.

Solution:

$$(a) \quad I_1 = \frac{V_8}{R_1 + R_2} = \frac{100}{(1 + 2.5) \times 10^3} = 28.57 \times 10^{-3} = 28.570 \text{ millamps}$$

$$I_2 = \frac{V_8}{R_3 + R_4} = \frac{100}{(6 + 3) \times 10^3} = 11.11 \times 10^{-3} = 11.111 \text{ milliamperes}$$

$$V_1 = I_1 R_1 = 28.570 \times 10^{-3} \times 1 \times 10^3 = 28.570 \text{ Volts}$$

$$V_2 = I_1 R_2 = 28.570 \times 10^{-3} \times 2.5 \times 10^3 = 71.425 \text{ Volts}$$

$$V_3 = I_2 R_3 = 11.111 \times 10^{-3} \times 6 \times 10^3 = 66.667 \text{ Volts}$$

$$V_4 = I_2 R_4 = 11.111 \times 10^{-3} \times 3 \times 10^3 = 33.333 \text{ Volts}$$

A. Problems with Solutions (continued)

1. (con't)

(b) Summing Loop 1: $V_b + V_1 + V_2 \stackrel{?}{=} 0$; $-100 + 28.570 + 71.425 \stackrel{?}{=} 0$
 $0 = 0$

Summing Loop 2: $V_b + V_3 + V_4 \stackrel{?}{=} 0$; $-100 + 66.667 + 33.333 \stackrel{?}{=} 0$
 $0 = 0$

(c) $P_1 = V_1 I_1 = 28.570 \times 28.570 \times 10^{-3} = 0.816 \text{ Watts}$

$P_2 = V_2 I_1 = 71.425 \times 28.470 \times 10^{-3} = 2.040 \text{ Watts}$

$P_3 = V_3 I_2 = 66.667 \times 11.111 \times 10^{-3} = 0.741 \text{ Watts}$

$P_4 = V_4 I_2 = 33.333 \times 11.111 \times 10^{-3} = 0.370 \text{ Watts}$

(d) Total power from battery:

$P_T = P_1 + P_2 + P_3 + P_4 = 0.816 + 2.040 + 0.741 + 0.370 = 3.967 \text{ Watts}$

Alternate proof of power delivered:

$P_T = V_b \times (I_1 + I_2) = 100 \times (28.570 \times 10^{-3} + 11.111 \times 10^{-3}) =$
 3.967 Watts

B. Problems without Solutions

2. The references represent two of the most important equations in electronics. Ref 1 defines the energy (power) being consumed in a circuit and is called Watt's Law. Ref 2 relates the voltage and current of a circuit to the circuit resistance and is the famous OHM's Law. It is common to use these equations in all of their possible forms.

(a) Rearrange OHM's Law to express I in terms of V and R and to express R in terms of V and I.

(b) Rearrange Watt's Law to express V in terms of P and I and to express I in terms of P and V.

(c) Combine OHM's Law and Watt's Law to express P in terms of I and R and P in terms of V and R.

Ref 1: $P = V \times I$ V in Volts; I in Amperes; P in Watts

2: $V = IR$ V in Volts; I in Amperes; R in OHMs

SIMPLE EQUATIONS
Level 2

Electronics

B., Problems without Solutions (continued)

2. (con't)

3. A radio amateur has calculated the inductance L needed to resonate with a 50 micromicrofarad capacitor at a frequency of 14.175 megahertz to be 2.52 microhenrys. He now wishes to wind that inductor as an air core coil with a diameter of 2 inches and a length of 6 inches. The reference gives a close approximation of the inductance of air core coils and is useable in this instance. How many turns will be required for the 2.52 microhenrys? Round to the nearest turn.

Ref: $L \text{ (microhenrys)} = \frac{a^2 n^2}{9a + 10b}$ where: a = radius of the coil (inches), b = length of the coil (inches), and n = number of turns.

B. Problems without Solutions (continued)

4. The resonant frequency of a capacitor in parallel with an inductor is given by the reference. Resonance is the natural frequency of exchange of electrical energy from the capacitor to the inductor and vice versa.

An amateur operator has on hand a capacitor of 50 micromicrofarads and wishes to wind an inductor coil to resonate with the capacitor at a frequency of 14.175 megahertz (20-meter band). Find the inductance of the coil which is to be wound.

Ref: $f = \frac{1}{2\pi\sqrt{LC}}$ f , in Hertz, L in Henrys and C in Farads

Note: Micromicrofarads = Farads $\times 10^{-12}$

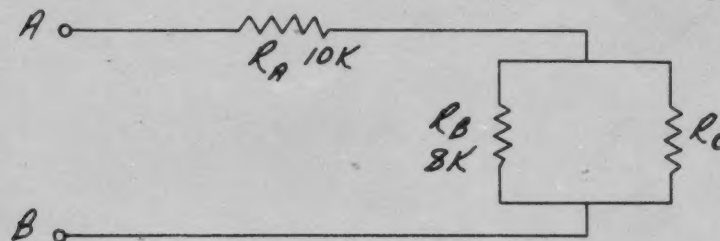
Megahertz = Hertz $\times 10^6$

B. Problems without Solutions (continued)

5. OHM's Law states that the resistance R of resistors in series adds directly while resistors in parallel combine according to

$$R_{\text{equivalent}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

Given the network below:



If an Ohmmeter at terminals A-B reads a total resistance of 13 K Ohms, what is the value in Kilohms R_C ? Round your answer to the nearest Kilohm.

B. Problems without Solutions (continued)

6. The resistance in Ohms of a conductor is given by:

$$R = p \frac{L}{A} \quad \text{where: } p = \text{coefficient of resistivity}$$
$$L = \text{length of conductor}$$
$$A = \text{cross-sectional area of conductor}$$

A communications technician is constructing a 5.0 Ohm power sink (to be used as a transmitter dummy load) from #16 Gage Nichrome resistance wire. Resistivity of Nichrome is 0.000113 Ohm-CM, and the diameter of the #16 gage wire is 0.129 CM. How many centimeters of the Nichrome wire will be needed? How many inches?

7. Radio waves travel at the speed of light (300 million meters/sec) in air, in space and in conductors. One WAVELENGTH, λ , is the distance a wave travels during the time for one cycle of the signal. The time for one cycle is determined by $T = \frac{1}{f}$ where T is in seconds, and f is in Hertz (cycles per second).

A radio technician wishes to design a vertical antenna, one-half wavelength long, for use at a frequency of 28.5 Megahertz.

- (a) Determine the length of the antenna in meters.
(b) What is the length in feet?

Note: Mega = Million = 10

SIMPLE EQUATIONS
Level 2

Electronics

Complete Solutions to B Problems

2. (a) $V = I R$ To solve for I , divide both sides by R :

$$\frac{V}{R} = I \quad \text{To solve for } R, \text{ divide both sides by } I: \frac{V}{I} = R$$

- (b) $P = V I$ To solve for V , divide both sides by I : $\frac{P}{I} = V$

$$\text{To solve for } I, \text{ divide both sides by } V: \frac{P}{V} = I$$

- (c) $P = V I$ Substitute for V : $P = (IR)(I) = I^2 R$
So $P = I^2 R$

$$\text{Substitute for } I: P = (V) \frac{V}{R} = \frac{V^2}{R}$$

$$\text{So } P = \frac{V^2}{R}$$

3. Solve for n :

$$L = \frac{a^2 n^2}{9a + 10b}$$

$$(L)(9a + 10b) = a^2 n^2$$

$$\frac{(L)(9a + 10b)}{a^2} = n^2$$

$$\sqrt{\frac{(L)(9a + 10b)}{a^2}} = n \quad \text{Now insert known values}$$

$$n = \sqrt{\frac{(2.52) [(9)(1) + (10)(6)]}{1^2}}$$

$$= 13.19$$

$$= 13 \text{ turns}$$

4. Solve the equation for L :

$$f = \frac{1}{2 \pi \sqrt{LC}}$$

$$\sqrt{LC} = \frac{1}{2 \pi f_0}$$

SIMPLE EQUATIONS
Level 2

Electronics

Complete Solutions to B Problems (continued)

4. (con't)

$$LC = \left(\frac{1}{2 \pi f_c} \right)^2$$

$$LC = \frac{1}{4 \pi^2 f_c^2}$$

$$L = \frac{1}{4 \pi^2 f_c^2 C}$$

$$L = \frac{1}{4 \pi^2 f_c^2 C} \quad \text{Now insert known values}$$

$$L = \frac{1}{(4 \pi^2) (14.175 \times 10^6 \text{ Hertz})^2 (50 \times 10^{-14} \text{ farads})}$$

$$L = \frac{1}{(4 \pi^2) (2.01 \times 10^8 \text{ H}) (50 \times 10^{-14} \text{ f})}$$

$$L = \frac{1}{3.97 \times 10^9}$$

$$L = 2.52 \times 10^{-6}$$

$$L = 2.52 \text{ MicroHenrys}$$

5. Set up the equation for the resistance of the network:

$$R_{\text{total}} = R_a + R_{b,c} = R_a + \frac{1}{\frac{1}{R_b} + \frac{1}{R_c}} = R_a + \frac{R_b R_c}{R_b + R_c}$$

Substituting known values (all resistances in Kilohms):

$$13 = 10 + \frac{8 R_c}{8 + R_c}$$

Clearing the denominator of R_c :

$$(13)(8 + R_c) = (10)(8 + R_c) + 8 R_c \text{ and } 104 + 13 R_c = 80 + 10 R_c + 8 R_c$$

Collecting terms and simplifying:

$$5 R_c = 24$$

$$R_c = 4.8 \text{ KOHms} \approx 5 \text{ KOHms}$$

Complete Solutions to B Problems (continued)

6. Computing the cross-sectional area of the wire:

$$A = \pi r^2 = \pi \left(\frac{d}{2} \right)^2 = \frac{\pi d^2}{4} = \frac{3.1416 \times (0.129 \text{ cm})^2}{4} = 0.013 \text{ cm}^2$$

Solving for L in cm

$$L = \frac{AR}{P} = \frac{0.012 \text{ cm}^2 \times 5 \text{ Ohm}}{0.000113 \text{ Ohm cm}} = 575.2 \text{ cm}$$

Converting to inches

$$L = 575.2 \text{ cm} \times 1 \text{ in}/2.54 \text{ cm} = 226.5 \text{ in}$$

7. (a) Find T, the time for one cycle:

$$T = \frac{1}{f} = \frac{1}{28.5 \times 10^6} = 0.0351 \times 10^{-6} = 3.51 \times 10^{-8} \text{ sec}$$

Find one wavelength of the wave

$$\left(\frac{3 \times 10^8 \text{ meters}}{\text{sec}} \right) \left(\frac{3.51 \times 10^{-8} \text{ sec}}{\text{Cycle}} \right) = 10.53 \text{ meters/cycle}$$

$$L = \frac{\lambda}{2} = 5.27 \text{ meters}$$

- (b) Convert $\lambda/2$ to feet:

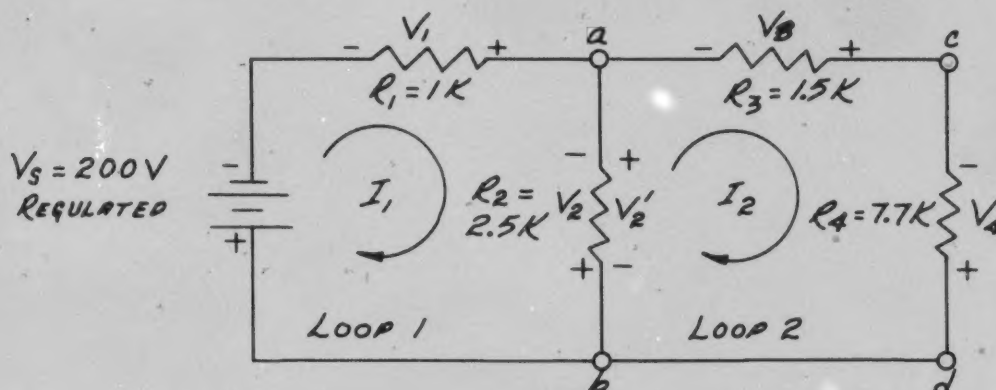
$$5.27 \text{ meters} \times \frac{3.28 \text{ ft}}{\text{meter}} = 17.3 \text{ feet}$$

$$L = 17.3 \text{ feet}$$

A. Problems with Solutions

1. Kirchoff's Voltage Law states that the algebraic sum of the voltage around any closed circuit loop is equal to zero.

The engineer has designed the double voltage-divider circuit below, and you are to breadboard and test it. Note that the battery symbol represents a regulated power supply with zero internal resistance.



Summing the two loops according to Kirchoff's Voltage Law,

$$\text{Loop 1: } -V_1 - V_2 + V_2' + 200 = 0$$

$$\text{Loop 2: } -V_3 - V_4 - V_2' + V_2 = 0$$

Note: With V in volts and R in KOhms, current is solved for in milliamps.

Using OHM's Law to define V in terms of I R,

$$V_1 = I_1 R_1; V_2 = I_2 R_2; V_2' = I_2 R_2; V_3 = I_2 R_3; V_4 = I_2 R_4$$

- (a) Substitute the IR products for the V's in the loop equations, and solve for I_1 and I_2 . Leave R in KOhms and I will then be in milliamps.
- (b) If the engineer expected -150V across points a-b and -111V across points c-d, were his calculations correct?

Solution:

$$\begin{aligned} \text{(a) Loop 1: } & -I_1 R_1 - I_1 R_2 + I_2 R_2 + 200 = 0 \\ & I_2 R_2 + 200 = I_1 R_1 + I_1 R_2 \\ & I_2 R_2 + 200 = I_1 (R_1 + R_2) \\ & 200 = I_1 (R_1 + R_2) - I_1 R_2 \end{aligned}$$

$$\begin{aligned} \text{Loop 2: Equation 1: } & -I_2 R_3 - I_2 R_4 + I_1 R_2 = 0 \end{aligned}$$

SIMULTANEOUS EQUATIONS
Level 2

Electronics

A. Problems with Solutions (continued)

1. (con't)

$$I_1 R_1 = I_2 R_3 + I_2 R_4 + I_2 R_2$$

$$I_1 R_1 = I_2 (R_3 + R_4 + R_2)$$

Equation 2:

$$\text{From equation 1, } 200 = I_1 (1 + 2.5) - I_2 (2.5)$$

$$200 + 2.5 I_2 = 3.5 I_1$$

$$I_1 = \frac{200 + 2.5 I_2}{3.5}$$

$$\text{Substituting into equation 2: } \left(\frac{200 + 2.5 I_2}{3.5} \right) (2.5) = I_2 (1.5 + 7.7 + 2.5)$$

$$\left(\frac{200 + 2.5 I_2}{3.5} \right) = \frac{I_2 (11.7)}{2.5}$$

$$200 + 2.5 I_2 = \frac{3.5 I_2 (11.7)}{2.5}$$

$$200 + 2.5 I_2 = 1.4 I_2 (11.7)$$

$$200 + 2.5 I_2 = 16.38 I_2$$

$$200 = 16.38 I_2 - 2.5 I_2$$

$$200 = I_2 (16.38 - 2.5)$$

$$200 = I_2 (13.88)$$

$$\frac{200}{13.88} = I_2$$

$$14.41 \text{ ma} = I_2$$

Substituting this value into equation 2 again,

$$I_1 (2.5) = 14.41 \text{ ma} (11.7)$$

$$I_1 = \frac{14.41 \text{ ma} (11.7)}{2.5}$$

$$I_1 = 67.44 \text{ ma}$$

(b) Calculating V (leave I in ma and R in Kilohms giving V in volts);

$$V_{ab} = -I_1 R_A + I_2 R_2 = -(67.4 \times 2.5) + (14.4 \times 2.5)$$

$$= -168.5 + 36$$

$$= -132.5 \text{ volts}$$

$$\text{Calculating } V_{cd}: V_{cd} = -I_2 R_4$$

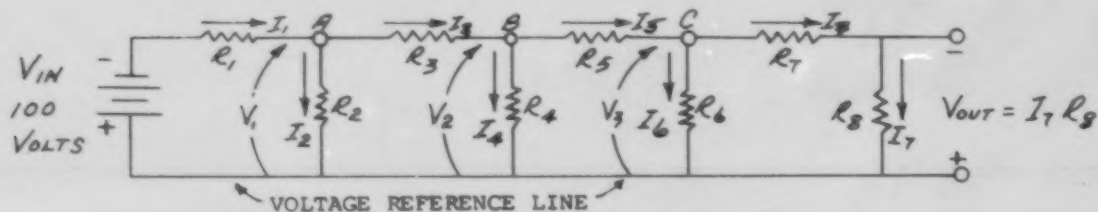
$$= -(14.4 \times 7.7)$$

$$= -111 \text{ volts}$$

So, for V_{ab} the engineer was wrong. For V_{cd} , he was right.

B. Problems without Solutions

2. In many cases a modified use of Kirchhoff's current law simplifies analysis of multiple-loop circuits. This technique is called "Node Voltage Analysis" and requires that the currents at the circuit junctions (nodes) be expressed in terms of voltage and resistance. For instance, in the circuit below, the currents approaching and leaving the nodes A, B, C can be expressed as indicated below the diagram:



$$I_1 = \frac{V_{in} - V_1}{R_1}, I_3 = \frac{V_1 - V_2}{R_3}, I_5 = \frac{V_2 - V_3}{R_5}, I_7 = \frac{V_3}{R_1 + R_7}$$

Since the sum of the currents at each node must equate to zero, a set of simultaneous equations can be written:

A. $I_1 - I_2 - I_3 = 0$; or $\frac{V_{in} - V_1}{R_1} - \frac{V_1}{R_2} - \frac{V_1 - V_2}{R_3} = 0$ If $V_{in} = 100$ Volts and:

B. $I_3 - I_4 - I_5 = 0$; or $\frac{V_1 - V_2}{R_3} - \frac{V_1 - V_3}{R_5} - \frac{V_2}{R_4} = 0$

C. $I_5 - I_6 - I_7 = 0$; or $\frac{V_2 - V_3}{R_5} - \frac{V_3}{R_6} - \frac{V_3}{R_7 + R_8} = 0$

$R_1 = 1.0K\Omega$ $R_5 = 0.5K\Omega$
 $R_2 = 10.0K\Omega$ $R_6 = 7.2K\Omega$
 $R_3 = 2.2K\Omega$ $R_7 = 1.2K\Omega$
 $R_4 = 6.7K\Omega$ $R_8 = 10.0K\Omega$

(1) Solve A, B, and C for V_1 , V_2 , and V_3 . Note: Leave R in $K\Omega$ and I will be measured in milliamps.

(2) Find V_{out}

SIMULTANEOUS EQUATIONS
Level 3

Electronics

B. Problems without Solutions (continued)

2. (con't)

SIMULTANEOUS EQUATIONS
Level 3

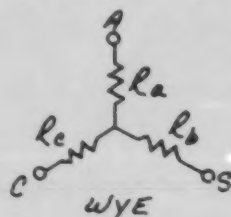
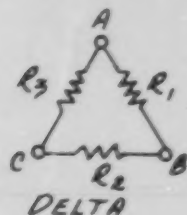
Electronics

B. Problems without Solutions (continued)

2. (con't)

B. Problems without Solutions (continued)

3. The "WYE" - "DELTA" transformation is used quite often in converting resistive networks and transformer hook-ups. The idea is that a circuit involving resistance and/or impedance can be configured in a wye or delta, while still retaining the terminal characteristics of the opposite arrangement. For instance, the two resistive networks below can have the same resistance as seen at terminals A-B, BC, CA if the proper values of resistance are chosen for each.



A set of simultaneous equations relating the two circuits can be written as follows:

$$\textcircled{1} R_a + R_b = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \quad \textcircled{2} R_b + R_c = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} \quad \textcircled{3} R_c + R_a = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

Solve and simplify the equations for R_a , R_b and R_c in terms of R_1 , R_2 and R_3 .

SIMULTANEOUS EQUATIONS
Level 3

Electronics

B. Problems without Solutions (continued)

3. (con't)

SIMULTANEOUS EQUATIONS
Level 3

Electronics

B. Problems without Solutions (continued)

3. (con't)

SIMULTANEOUS EQUATIONS
Level 3

Electronics

Complete Solutions to B Problems

2. (1) Substituting values into equations A,

$$\frac{100 - V_1}{1} - \frac{V_1}{10} - \frac{V_1 - V_2}{2.2} = 0$$

$$1000 - 10V_1 - V_1 - \frac{10(V_1 - V_2)}{2.2} = 0$$

$$2200 - 22V_1 - 2.2V_1 - 10V_1 + 10V_2 = 0$$

$$2200 + 10V_2 = 22V_1 + 2.2V_1 + 10V_1$$

$$2200 + 10V_2 = 34.2V_1$$

$$10V_2 = 34.2V_1 - 2200$$

$$V_2 = 3.42V_1 - 220$$

or

$$\frac{V_2 + 220}{3.42} = V_1$$

$$\begin{aligned} R_1 &= 1 \\ R_2 &= 10 \\ R_3 &= 2.2 \\ R_4 &= 6.7 \\ R_5 &= 0.5 \\ R_6 &= 7.2 \\ R_7 &= 1.2 \\ R_8 &= 10 \end{aligned}$$

Substituting this value for V into equation B,

$$\frac{V_2 + 220}{3.42} - \frac{V_2}{2.2} - \frac{V_2 - V_3}{0.5} - \frac{V_2}{6.7} = 0$$

$$\frac{V_2 + 220}{3.42} - V_2 - \frac{2.2(V_2 - V_3)}{0.5} - \frac{2.2(V_2)}{6.7} = 0$$

$$\frac{V_2 + 220}{3.42} - V_2 - 4.4(V_2 - V_3) - 0.33(V_2) = 0$$

$$V_2 + 220 - 3.42V_2 - 15.05(V_2 - V_3) - 1.13(V_2) = 0$$

$$V_2 + 220 - 3.42V_2 - 15.05V_2 + 15.05V_3 - 1.13V_2 = 0$$

$$15.05V_3 = 3.42V_2 + 1.13V_2 + 15.05V_2 - V_2 - 220$$

$$15.05V_3 = 18.6V_2 - 220$$

$$15.05V_3 + 220 = 18.6V_2$$

$$\frac{15.05V_3 + 220}{18.6} = V_2$$

Substituting this value for V in equation C,

$$\frac{15.05V_3 + 220}{18.6} - V_3 - \frac{V_3}{7.2} - \frac{V_3}{11.2} = 0$$

SIMULTANEOUS EQUATIONS
Level 3

Electronics

Complete Solutions to B Problems

2. (con't)

$$\frac{30.1V_3 + 440}{18.6} - 2V_3 - V_3/7.2 - V_3/11.2 = 0$$

$$1.62V_3 + 23.66 - 2V_3 - 0.14V_3 - 0.09V_3 = 0$$

$$1.62V_3 - 2V_3 - 0.14V_3 - 0.09V_3 = -23.66$$

$$-0.61V_3 = -23.66$$

$$V_3 = 38.79 \text{ volts}$$

$$\text{Above, we found } \frac{15.05V_3 + 220}{18.6} = V_2$$

$$\text{Substituting, } \frac{(15.05)(38.79) + 220}{18.6} = 43.21 \text{ Volts} = V_2$$

$$\text{From equation A, we found } \frac{V_2 + 220}{3.42} = V_1$$

$$\text{Substituting, } \frac{43.21 + 220}{3.42} = 76.96 \text{ Volts} = V_1$$

(2) V_{out} from the diagram is $(I_7)(V_7)$

$$I_7 \text{ is given as } \frac{V_3}{R_1 + R_4}$$

$$\text{Substituting, } \frac{38.79}{11.2} = 3.46 \text{ milliamps} = I_7$$

$$\text{So, } (I_7)(V_8) = (3.46 \text{ milliamps})(10K \Omega) = 34.6 \text{ Volts}$$

3. The three equations are: 1 $R_a + R_b = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$

2 $R_b + R_c = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3}$

3 $R_c + R_a = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$

$$\text{From equation 1 } R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - R_b$$

$$\text{But from equation 2 } R_b = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - R_c$$

$$\text{And from 3 } R_c = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - R_a$$

Complete Solutions to B Problems (continued)

3. (con't)

$$\begin{aligned} \text{So, } R_a &= \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - R_c \\ &= \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - \left(\frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - \left[\frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - R_a \right] \right) \end{aligned}$$

$$R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} + \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - R_a$$

$$2R_a = \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3} - \frac{R_2R_1 + R_2R_3}{R_1 + R_2 + R_3} + \frac{R_3R_1 + R_3R_2}{R_1 + R_2 + R_3}$$

$$2R_a = \frac{2(R_1R_3)}{R_1 + R_2 + R_3}$$

$$R_a = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

$$\begin{aligned} R_b &= \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - R_c \\ &= \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - \left(\frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - R_a \right) \\ &= \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - \left(\frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - \left[\frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - R_b \right] \right) \end{aligned}$$

$$R_b = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} + \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - R_b$$

$$2R_b = \frac{R_2R_1 + R_2R_3}{R_1 + R_2 + R_3} - \frac{R_3R_1 + R_3R_2}{R_1 + R_2 + R_3} + \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3}$$

$$2R_b = \frac{2(R_2R_1)}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2R_1}{R_1 + R_2 + R_3}$$

$$\begin{aligned} R_c &= \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - R_a \\ &= \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - \left(\frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - R_b \right) \end{aligned}$$

SIMULTANEOUS EQUATIONS
Level 3

Electronics

Complete Solutions to B Problems (continued)

3. (con't)

$$R_C = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - \left(\frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} - \left[\frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - R_C \right] \right)$$
$$= \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} - \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} + \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} - R_C$$

$$2R_C = \frac{R_3R_1 + R_3R_2}{R_1 + R_2 + R_3} - \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3} + \frac{R_2R_1 + R_2R_3}{R_1 + R_2 + R_3}$$

$$2R_C = \frac{2(R_1R_3)}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

A. Problems with Solutions

1. In complex filter circuits it is very often possible to locate the frequencies of series resonance (Zero impedance) and parallel resonance (Infinite impedance) through a technique called "Poles" and "Zeros". In applying this approach, we first solve for the equations representing circuit impedance Z as seen from the input terminals. This equation, for some simple circuits, is a fraction with a quadratic equation in the denominator.

The impedance of a capacitor in parallel with an inductor is shown below:

$$Z = \frac{2\pi fL}{4\pi^2 f^2 LC - 1} \quad \begin{array}{l} L = \text{Inductance; } C = \text{Capacitance} \\ f = \text{frequency} \end{array}$$

If frequency is the only variable, locate the frequencies for Zero impedance (numerator = 0) and for Infinite impedance (denominator = 0). Note that negative frequency has no meaning.

Solution:

$$Z = \frac{2\pi fL}{4\pi^2 LC f^2 - 1}$$

Locating the "zero's" (numerator = 0),

$2\pi fL = 0$; since f is the only variable, the zero is found at $f = 0$ for $Z_T = 0$

Locating the "poles" (denominator = 0),

$$4\pi^2 LC f^2 - 1 = 0$$

Factoring the quadratic and equating to zero,

$$[(\sqrt{4\pi^2 LC})f - 1][(\sqrt{4\pi^2 LC})f + 1] = 0 \text{ two roots ("poles") exist}$$

Thus,

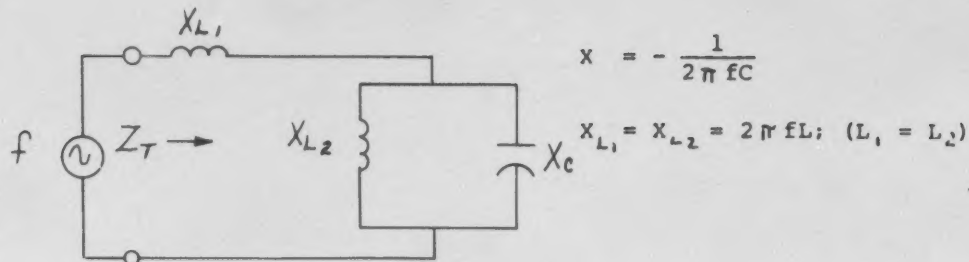
$$(\sqrt{4\pi^2 LC})f_1 - 1 = 0; \quad (\sqrt{4\pi^2 LC})f_2 + 1 = 0$$

$$f_1 = \frac{1}{2\pi\sqrt{LC}}; \quad f_2 = -\frac{1}{2\pi\sqrt{LC}} \quad \text{But since negative } f \text{ has no meaning,}$$

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{for } Z_T = \infty$$

B. Problems without Solutions

2.



For the series-parallel filter arrangement above, the impedance Z seen by the generator is a function of frequency. The frequencies of interest are those producing series resonance ($Z = 0$) or parallel resonance ($Z_T = \infty$). Solving this network for Z yields:

$$Z = \frac{X_L^2 + 2X_L X_C}{X_L + X_C}$$

- (a) Assuming $X_L + X_C$ cannot reach infinity, find X_L in terms of X_C at the points of series resonance (numerator = 0).
- (b) Find the relationship of X_L and X_C at parallel resonance (denominator = 0).
- (c) If $L_1 = L_2 = 10$ millihenrys and $c = 0.025$ microfarads, find the frequencies of series and parallel resonance.

Complete Solutions to B Problems

2. (a) For series resonance, $Z_T = 0$ and $X_L^2 + 2X_L X_C = 0$

$$\text{Factoring out } X_L, \quad X_L (X_L + 2X_C) = 0$$

$$X_L = 0 \text{ and } X_L = -2X_C$$

- (b) For parallel resonance, $Z_T = \infty$ and $X_L + X_C = 0$ (a limit condition)

$$X_L = -X_C$$

- (c) Series resonance:

For $X_L = 0$; $2\pi fL = 0$ therefore, $f = 0$ (direct current-not resonance)

For $X_L = -2X_C$ substituting values and solving for f :

$$2\pi fL = -2 \left(-\frac{1}{2\pi fC} \right); \quad 2\pi f(10 \times 10^{-3} \text{ henrys}) = -2 \left[-\frac{1}{2\pi f(0.025 \times 10^{-6} f)} \right]$$

$$f = \sqrt{\frac{2}{4\pi^2(10 \times 10^{-3} \text{ henrys})(0.025 \times 10^{-6} \text{ farads})}} = 14.2 \text{ KHz}$$

Parallel resonance:

$$X_L = -X_C; \quad 2\pi f(10 \times 10^{-3} \text{ henrys}) = \frac{1}{2\pi f(0.025 \times 10^{-6} \text{ farads})}$$

$$f = \sqrt{\frac{1}{4\pi^2(10 \times 10^{-3} \text{ henrys})(0.025 \times 10^{-6} \text{ farads})}} = 10 \text{ KHz}$$

SIMPLE EQUATIONS
Level 1

Marketing

A. Problems with Solutions

1. Mr. Goodman invests part of \$7,500 at 3% annual interest and the rest at 5% annual interest. The income from the two investments is \$279 annually. How much did Mr. Goodman invest at each rate?

Solution:

List the quantities you will use:

Amount invested at 3% = x dollars
Amount invested at 5% = $7,500 - x$ dollars
Income from the 3% investment = $0.03x$ dollars
Income from the 5% investment = $0.05(7,500 - x)$ dollars
Income from both investments = \$279

$$0.03x + (0.05)(7,500 - x) = \$279$$

$$0.03x + \$375 - 0.05x = \$279$$

$$0.03x - 0.05x = \$279 - \$375$$

$$-0.02x = -\$96$$

$$x = \frac{-\$96.00}{-0.02} = \$4,800$$

$$\text{Amount invested at 5\%} = \$7,500 - \$4,800 = \$2,700$$

$$\text{Amount invested at 3\%} = \$4,800$$

2. Mike and Bob are partners. The agreement is that Mike's profit is to be 25% greater than that of Bob. The profit to be divided for this year is \$52,000. How much did each partner receive?

Solution:

x = amount Bob receives

$x + 0.25x$ = amount Mike receives

$$x + x + 0.25x = \$52,000$$

$$2.25x = \$52,000$$

$$x = \frac{\$52,000}{2.25} = \$23,111.11$$

$$\begin{aligned} x + 0.25x &= \$23,111.11 + \$5,777.78 \\ &= \$28,888.89 \end{aligned}$$

Mike receives \$28,888.89

Bob receives \$23,111.11

SIMPLE EQUATIONS
Level 1

Marketing

A. Problems with Solutions (continued)

3. A suit cost \$65. What must the suit be sold for if a profit of 20% of the selling price is to be made?

Solution:

$$\text{Profit} = (0.20)(\text{selling price})$$

$$\text{Selling price} = \text{cost} + \text{profit}$$

$$\text{SP} = \text{Selling price}$$

$$\text{P} = \text{profit}$$

$$\text{SP} = \$65.00 + \text{P} \quad \text{P} = (0.20)\text{SP}$$

$$\text{SP} = \$65.00 + (0.20)\text{SP}$$

$$(0.80)\text{SP} = \$65.00$$

$$\text{SP} = \frac{\$65.00}{0.80} = \$81.25$$

4. How much will Tina, Shelley and Connie each receive if \$180 is to be divided so that Shelley receives $\frac{2}{3}$ as much as Tina, and Connie receives $\frac{1}{2}$ as much as Shelley?

Solution:

$$x = \text{amount that Tina receives}$$

$$(\frac{2}{3})x = \text{amount that Shelley receives}$$

$$\frac{1}{2}(\frac{2}{3})x = \text{amount that Connie receives}$$

$$x + (\frac{2}{3})x + \frac{1}{2}(\frac{2}{3})x = \$180$$

$$x + \frac{2}{3}x + \frac{1}{3}x = \$180$$

$$2x = \$180$$

$$x = \$90$$

$$\frac{2}{3}x = (\frac{2}{3})(\$90) = \$60$$

$$\frac{1}{2}(\frac{2}{3})x = (\frac{1}{2})(\$60) = \$30$$

Tina receives \$90, Shelley receives \$60 and Connie receives \$30.

SIMPLE EQUATIONS
Level 1

Marketing

B. Problems without Solutions

5. An automobile costs a dealer \$1,300. What must his selling price be to give him 20% of the selling price as overhead and 15% of the selling price for profit?

6. When an article costs \$6, what should the minimum selling price be so that the profit will be at least 20% of the selling price?

7. One card sorter can process a deck of punched cards in 30 minutes while another can sort the deck in 45 minutes. How long would it take the two sorters together to process the cards?

B. Problems without Solutions (continued)

8. In a factory, one worker can complete a job with a punch press in 7 days, but it takes another worker 8 days to complete the same job with the same kind of machine. If both machines are operating at the same time, how long will it take to do this job?

9. A grocer mixed nuts selling at 40¢ a pound with some selling for 60¢ a pound. If he got 200 lb which he could sell for 48¢ a pound, how many pounds of the 60¢ nuts did he use?

10. A discount store owner pays \$15 for a transistor radio on which he wishes to make a 30% profit based on the selling price after he has allowed a discount of 10% to his customer. What should he set as the list price of the radio?

SIMPLE EQUATIONS
Level 1

Marketing

Complete Solutions to B Problems

5. SP = Selling Price

$$\text{Overhead} = (0.20)\text{SP}$$

$$\text{Profit} = (0.15)\text{SP}$$

$$\text{SP} = \text{Cost} + \text{Profit} + \text{Overhead}$$

$$\text{SP} = \$1,300 + (0.15)\text{SP} + (0.20)\text{SP}$$

$$\text{SP} - (0.15)\text{SP} - (0.20)\text{SP} = \$1,300$$

$$(0.65)\text{SP} = \$1,300$$

$$\text{SP} = \frac{\$1,300}{0.65} = \$2,000$$

6. SP = Selling Price

$$\text{Profit} = (0.20)\text{SP}$$

$$\text{SP} = \text{cost} + \text{profit}$$

$$\text{SP} = \$6.00 + (0.20)\text{SP}$$

$$\text{SP} - (0.20)\text{SP} = \$6.00$$

$$(0.80)\text{SP} = \$6.00$$

$$\text{SP} = \frac{\$6.00}{0.80} = \$7.50$$

7. Let x = minutes required with both sorters working

Then $\frac{1}{x}$ = part of the job that can be done in one minute when sorters are sorting.

To help see this, note that:

$$\text{Whole job} = (\text{minutes worked}) (\text{part of job per minute})$$

$$= (x) \left(\frac{1}{x} \right)$$

$$= 1$$

$\frac{1}{45}$ = part of job 45 min sorter does in 1 minute.

$\frac{1}{30}$ = part of job 30 min sorter does in 1 minute

SIMPLE EQUATIONS
Level 1

Marketing

Complete Solutions to B Problems (continued)

7. (continued)

$$\frac{1}{45} + \frac{1}{30} = \text{part of job both can do in 1 minute}$$

$$\text{Thus, } \frac{1}{45} + \frac{1}{30} = \frac{1}{x}$$

Multiply both sides by (30)(45)(x):

$$(30)(45) \left[\frac{1}{45} + \frac{1}{30} \right] = \left(\frac{1}{x} \right) (30)(45)(x)$$

$$30x + 45x = 1,350$$

$$75x = 1,350$$

$$x = \frac{1,350}{75} = 18 \text{ min}$$

8. (Very similar to problem 7.)

Let x = time required with both machines running

$$\text{then } \frac{1}{7} + \frac{1}{8} = \frac{1}{x} \quad (\text{see discussion in problem 7})$$

multiply by (7)(8)(x):

$$(7)(8)(x) \left[\frac{1}{7} + \frac{1}{8} \right] = \frac{1}{x} (7)(8)(x)$$

$$8x + 7x = 56$$

$$15x = 56$$

$$x = \frac{56}{15} = 3 \frac{11}{15} \text{ days}$$

9. x = pounds of 60¢ nuts
 $200 - x$ = pounds of 40¢ nuts

$(\$0.60)x$ = income from selling x lbs at 60¢ a pound

$(\$0.40)(200 - x)$ = income from selling $200 - x$ lbs at 40¢ a pound

$(\$0.48)(200)$ = income from selling 200 lbs at 48¢ a pound

SIMPLE EQUATIONS
Level 1

Marketing

Complete Solutions to B Problems (continued)

9. (continued)

The income from the sale of the 48¢ mixture must equal the combined income from the sale of the 60¢ and 40¢ nuts.

$$(\$0.40/\text{lb})(200 \text{ lbs} - x) + (\$0.60/\text{lb})(x) = (\$0.48/\text{lb})(200 \text{ lbs})$$

$$\$80.00 - (\$0.40/\text{lb})x + (\$0.60/\text{lb})x = \$96.00$$

$$(\$0.20/\text{lb})x = \$96.00 - \$80.00$$

$$(\$0.20/\text{lb})x = \$16.00$$

$$x = \frac{\$16.00}{\$0.20/\text{lb}} = 80 \text{ lbs}$$

10. List Price = LP
Selling Price = SP

$$LP = \$21.43 + (0.10)LP$$

$$LP = SP + \text{Discount}$$

$$(0.90)LP = \$21.43$$

$$\text{Discount} = (0.10)LP$$

$$LP = \frac{\$21.43}{0.90} = \$23.81$$

$$SP = \text{cost} + \text{profit}$$

$$\text{Profit} = (0.30)SP$$

$$SP = \$15.00 + (0.30)SP$$

$$(0.70)SP = \$15.00$$

$$SP = \frac{\$15.00}{0.70} = \$21.43$$

A. Problems with Solutions

1. A teller mistakenly reversed the two digits in the face amount of a check, overpaying \$9. If the sum of the digits was 9, determine the amount for which the check was drawn. (Assume the check amount was between \$10 and \$99.)

Solution:

Let x = first digit in correct number
 y = second digit in correct number

Then, $10x + y$ = correct check amount

$10y + x$ = incorrect check amount

Since the teller overpaid \$9

$$10x + y + 9 = 10y + x$$

$$10x - x + y - 10y = -9$$

$$9x - 9y = -9$$

$$x - y = -1$$

$$x = y - 1$$

It was given that $x + y = 9$

$$\text{So, } \left. \begin{array}{l} x - y = -1 \\ x + y = 9 \end{array} \right\} \text{ are the two equations}$$

$$\text{Since } x = y - 1 \text{ + } x + y = 9$$

$$(y-1) + y = 9$$

$$2y - 1 = 9$$

$$2y = 10$$

$$y = 5$$

$$x = 4$$

Correct check amount = \$45

SYSTEMS OF EQUATIONS
Level 1

Marketing

A. Problems with Solutions (continued)

2. A grocer blends teas worth 66¢ and 48¢ a pound. If he interchanges the amounts, he saves \$6 in a blend of 100 pounds. Find the ratio of the weight of the two teas in the original blend.

Solution:

Let x = lbs of 66¢ tea in original blend

y = lbs of 48¢ tea in original blend

Then $66x + 48y$ = cost in cents of original blend

$66y + 48x$ = cost in cents of original blend

Since the total weight was 100 lbs

$$x + y = 100$$

$$x = 100 - y$$

Since the new blend saves 600¢ on the original

$$66x + 48y = 66y + 48x + 600$$

$$66(100-y) + 48y = 66y + 48(100-y) + 600$$

$$6600 - 66y + 48y = 66y + 4800 - 48y + 600$$

$$6600 - 18y = 18y + 5400$$

$$6600 = 36y + 5400$$

$$1200 = 36y$$

$$33 \frac{1}{3} = y$$

From above

$$x + 33 \frac{1}{3} = 100$$

$$x = 66 \frac{2}{3}$$

$$\text{Ratio of 66¢ to 48¢ teas} = \frac{66 \frac{2}{3}}{33 \frac{1}{3}} = 2/1 \text{ or } 2 \text{ to } 1$$

A. Problems with Solutions (continued)

3. In two years, a sum of money at simple interest amounted to \$330 while in five years it amounted to \$375. What was the sum, and what was the interest rate?

Solution:

Let P = the original sum (principal)

R = interest rate

Note: A basic formula used in this situation is $I = P R T$

where I = interest earned in T years on the principal (P)

Therefore, we have:

$$\$330 = P R 2 + P$$

$$\$375 = P R 5 + P$$

or factoring:

$$\$330 = P(2R+1)$$

$$\$375 = P(5R+1)$$

$$\text{From the first equation } P = \frac{\$330}{2R+1}$$

Substituting in the second equation

$$\$375 = \frac{\$330}{2R+1} (5R+1)$$

$$\$375(2R+1) = \$330(5R+1)$$

$$750R + 375 = 1650R + 330$$

$$375 = 900R + 330$$

$$45 = 900R$$

$$\frac{45}{900} = R$$

$$.05 = R$$

$$\text{From above } P = \frac{\$330}{2(.05)+1}$$

$$P = \frac{\$330}{1.1}$$

$$P = \$300$$

4. Working together, two sorters can process 1,200 cards in one minute. The faster sorter breaks down after operating $\frac{1}{2}$ minute, and the time to process 1,200 cards is increased to 2 minutes. How many cards can each sorter handle in one minute?

5. Payless makes a profit of 2¢ on each bottle of brand X aspirin that it sells, and a profit of 6¢ on each bottle of brand Y aspirin that it sells. How many bottles of each brand must the druggist sell if he is to make a total profit of \$8 from the sales of these two brands of aspirin?
6. Refer to problem 5 and decide how many bottles of each brand the druggist must sell if the total number of bottles sold is 280.

B. Problems without Solutions (continued)

7. The butcher is to buy some chickens and ducks. He will sell the chickens at a profit of 2¢ per lb and will lose 1¢ per lb on the ducks. He does not wish to make a profit since these items will be used to attract customers. Nor does he intend to lose money on the sales. The suppliers will sell to him at attractive prices only if he buys the same number of pounds of ducks as of chickens. How many pounds of each should the butcher buy in order to meet these conditions.

8. The same butcher decides to make a profit of \$20 on the sale of chickens and ducks. The market has changed, and now he must buy 100 lbs more of ducks than of chickens. How many pounds of each should he buy?

B. Problems without Solutions (continued)

9. Suppose that the butcher now makes 2¢ per lb on chickens and loses 1¢ per lb on ducks. He wishes to make \$10 for handling costs. The supplier now requires that he buy in such a way that the pounds of chickens purchased exceed by 500 lbs one-half the number of pounds of ducks.
- (a) How many pounds of ducks should he buy?
 - (b) Chickens?
 - (c) If he buys 300 lbs of chickens, can he achieve his goal of \$10?
 - (d) If he buys 150 lbs of ducks, can he achieve his goal?

B. Problems without Solutions (continued)

10. A mail-order firm uses the following equation to estimate the number of dollars to sales for the day from the weight of first-class mail received by 7:00 a.m.

Sales = $A + B(x)$ where x is the weight of the mail.

If the number of shipping clerks needed is one per \$12,000 of sales, find the number of clerks needed on a day when the mail weighs 300 lbs given

$$11A + 115B = 49,620$$

$$115A + 1,595B = 683,700$$

SYSTEMS OF EQUATIONS
Level 1

Marketing

Complete Solutions to B Problems

4. Let x = number of cards slow sorter can handle in one minute
 y = number of cards fast sorter can handle in one minute

Two equations are:

$$\begin{aligned}x + y &= 1,200 \\ 2x + 1/2y &= 1,200\end{aligned}$$

From first equation
 $x = 1,200 - y$

Substituting into second equation
 $2(1,200 - y) + 1/2y = 1,200$

$$2,400 - 2y + 1/2y = 1,200$$

$$2,400 - 1\frac{1}{2}y = 1,200$$

$$1,200 = 1\frac{1}{2}y$$

$$800 = y \quad \text{cards per minute}$$

Since $x + y = 1,200$

$$x = 400 \text{ cards per minute}$$

5. Let x = number of bottles of brand x
 y = number of bottles of brand y

$$\begin{aligned}.02x &= \text{profit from sale of brand } x \\ .06y &= \text{profit from sale of brand } y\end{aligned}$$

The only equation obtained from this information is:

$$.02x + .06y = 8$$

Therefore, any x and y values satisfying this equation would be valid.

6. Using the same definitions as in solution 5, the two equations used are:

$$\begin{aligned}x + y &= 280 \\ .02x + .06y &= 8\end{aligned}$$

From equations 1, $x = 280 - y$

So, $.02(280 - y) + .06y = 8$

$$5.6 - .02y + .06y = 8$$

$$5.6 + .04y = 8$$

$$.04y = 2.40$$

$$y = 60 \text{ bottles}$$

$$\text{so } x = 220 \text{ bottles}$$

SYSTEMS OF EQUATIONS
Level 1

Marketing

Complete Solutions to B Problems (continued)

7. Let x = lbs of chickens
 y = lbs of ducks

$$\begin{aligned}.02x &= \text{profit on a chicken} \\ -.01y &= \text{profit on a duck (a loss)}\end{aligned}$$

The two equations are

$$\begin{aligned}.02x - .01y &= 0 \\ x - y &= 0\end{aligned}$$

Since $x = y$

$$.02x - .01x = 0$$

$$.01x = 0$$

$$x = 0$$

$$y = 0$$

8. Refer to problem 7

The two equations are now:

$$\begin{aligned}.02x - .01y &= 20 \\ x + 100 &= y\end{aligned}$$

$$\text{So, } .02x - .01(x + 100) = 20$$

$$.02x - .01x - 1 = 20$$

$$.01x = 21$$

$$\begin{aligned}x &= 2,100 \text{ lbs of chicken} \\ y &= 2,200 \text{ lbs of duck}\end{aligned}$$

9. Refer to problem 7

The two equations are now:

$$\begin{aligned}.02x - .01y &= 10 \\ x - 500 &= y/2\end{aligned}$$

$$\text{From equation \#2: } 2x - 1000 = y$$

SYSTEMS OF EQUATIONS
Level 1

Marketing

Complete Solutions to B Problems (continued)

9. (continued)

Substituting into equation 1:

$$.02x - .01(2x - 1000) = 10$$

$$.02x - .02x + 10 = 10$$

$$10 = 10$$

These are dependent equations. This means that x and y can be any numbers as long as they satisfy either of the equations:

$$x - 500 = y/2 \quad \text{or} \quad .02x - .01y = 10$$

(c) Set $x = 300$

This means

$$300 = 500 + y/2$$

$$600 = 1000 + y$$

$$-400 = y$$

Since y must be positive, this situation is impossible

(d) Set $y = 150$

$$\text{Then } x = 500 + 150/2$$

$$x = 500 + 75$$

$$x = 575$$

Therefore, he can achieve his goal

10. Solve the given system of equations for $A + B$

$$11A + 115B = 49,620$$

$$115A + 1595B = 683,700$$

From the first equation

$$A = \frac{49,620 - 115B}{11} \quad \text{substituting in equation 2}$$

$$115 \frac{49,620 - 115B}{11} + 1,595B = 683,700$$

$$115(4510.9 - 10.5B) + 1,595B = 683,700$$

$$392.7B = 164,946.5$$

$$B = 420$$

SYSTEMS OF EQUATIONS
Level 1

Marketing

Complete Solutions to B Problems (continued)

10. (continued)

$$A = \frac{49,620 - 115B}{11} = \frac{49,620 - 48,300}{11} = \frac{1,320}{11} = 120$$

$$\text{Sales} = A + B(x)$$

$$\begin{aligned} &= 120 + 420(300) \\ &= 120 + 126,000 \\ &= \$126,120 \end{aligned}$$

$$\begin{aligned} \text{Number of clerks} &= \frac{\text{total sales}}{\text{clerks/sale amount}} \\ &= \frac{\$126,120}{\$12,000/\text{clerks}} \end{aligned}$$

$$= 10.5 \text{ clerks}$$

$$= 11 \text{ clerks}$$

FORMULAS
Level I

Industrial, Electrical and
Hydraulics

A. Problems with Solutions

1. An electric range specification name plate gives the following information:

$$12\frac{3}{4} \text{ kW} \quad 230\text{V}$$

How many amperes does it draw?

Note: Electrical Power P, in watts, equals the product of the current I in amperes and the electromotive force E, in volts.

Solution: $12\frac{3}{4} \text{ kW} = 12,750 \text{ watts}$

$$\text{Since (EMF (in volts)) (current (in amperes)) = (Power (in watts))}$$

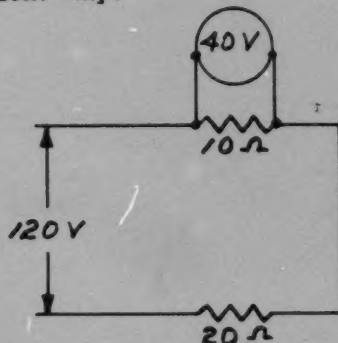
$$\text{Current} = \frac{\text{power}}{\text{EMF}}$$

$$= \frac{12,750 \text{ watts}}{230 \text{ volts}}$$

$$= 55.4 \text{ Amperes}$$

Level II

2. To make a test taking several hours and requiring about 40 volts, it is suggested that a 10 ohm and a 20 ohm resistor be connected in a series across a 120 volt source to give 40 volts across the 10 ohm resistor. The resistors are rated 200 watt. Is this a good suggestion? Why?



Solution: Since V drop across the 10 Ω resistor is 40 volts, the drop across the 20 Ω resistor is $120\text{V} - 40\text{V} = 80\text{V}$.

$$\text{Total resistance} = R_1 + R_2 = 10 + 20 = 30$$

$$\text{Current in the circuit is } \frac{120 \text{ volts}}{30 \Omega} = 4\text{A}$$

FORMULAS
Level I

Industrial, Electrical and
Hydraulics

A. Problems with solutions

2. cont'd Power = (voltage drop) (amperes)
Power in 10 Ω resistor = (40V) (4A) = 160 watts
Power in 20 Ω resistor = (80V) (4A) = 320 watts
Since these resistors were rated 200 watts this would
overload the 20 Ω resistor.

Level I

B. Problems without solutions

3. A meter repairman needs a shunt to carry a 25 ampere load at 50 millivolts. What resistance should the shunt have?

Note: Ohm's law states Resistance = $\frac{\text{voltage}}{\text{current}}$

Level II

4. The Electrical Code specifies that the maximum IR drop between a dc motor and its control switch is not to exceed 1% of the supply voltage. If the supply is 200 Volts and full load motor current is 100 amperes, the distance from the motor to its switch is 100 ft. and the resistivity of copper is 10 ohms per circular-mill foot, what size wire is required?

Note: Resistance = $\frac{\text{voltage}}{\text{current}}$

B. Problems without solutions

5. To obtain a permit to install an electric baseboard heating system, a permit application must be completed. The following copy of an application form has the surface areas and heat loss factors filled in.

You are to complete the following on the form.

- 1) On the "Room Heat Loss" part, calculate the BTU for each area given by multiplying each area by the corresponding heat loss factor.
- 2) Total the BTU loss for each room.
- 3) Calculate Installed Watts for each room.
 - a) $\text{Installed Watts} = \frac{\text{Total BTU loss}}{3.413}$ rounded up to
 - b) Minimum Installed Watts to be figured in Bath Room is 750.
 - c) Minimum Installed Watts for any other room is 500.
- 4) Calculate Installed BTU
 $\text{Installed BTU} = (\text{Installed watts}) \times (3.413)$
- 5) In the "Building Heat Loss" part of the form, calculate the heat loss in BTU per hour for each type of building material, ie; glass, doors, etc.
- 6) Find the total Building heat loss and add 10%.
This, BTU P/H + 10% must be equal to or less than the total installed BTU figured.
- 7) If BTU P/H + 10% from step 6 is greater than the total installed BTU calculated, increase the wattage of units in those rooms which show an installed BTU equal to or just slightly greater than the BTU loss for those rooms.

FORMULAS
Level II

Industrial, Electrical and
Hydraulics

B. Problems without solutions

5. continued

ROOM HEAT LOSS

Rooms & Size		LR- DR		Kitch-Nook		Utility		BR#1		BR#2		BR#3	
	FACTOR*	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU
WINDOWS	90	95		15		9		33		18		18	
Doors, Wood	80	20				40							
SLD- Glass	150			40									
Walls	4	373		105		59		223		78		78	
PARTITIONS	5			24		140							
CEILINGS	3	592		263		100		256		192		180	
FLOORS	5	592		263		100		256		192		180	
TOTAL BTU LOSS													
INSTALLED BTU													
INSTALLED WATTS													

I agree to install according to the Heating and Ventilating
Regulations of the City of

Rooms & Size		FAM-RM		EATH #1		BATH # 2		HALL					
	FACTOR*	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU
WINDOWS	90	18		6		9		0					
DOORS, WOOD	80												
SLD GLASS	150	40											
WALLS	4	102		50		55		0					
PARTITIONS													
CEILINGS	3	264		60		80		105					
FLOORS	5	264		60		80		105					
TOTAL BTU LOSS													
INSTALLED BTU													
INSTALLED WATTS													

I agree according to the Heating and Ventilating
Regulations of the City of

B. Problems without Solutions

6. An electric utility company representative prepares the following Heat Loss Estimate in order to estimate the approximate yearly cost to electrically heat a home. You are to complete the estimate as follows:
 1. Apply the heat loss factors, BTU/sq ft/hr, to the appropriate surface areas to find the heat loss in BTU/hr for each type of surface.
 2. Find the total heat loss calculated in BTU/hr for the building living area.
 3. Find the BTU/hr/sq ft average for the living area floor space.
 4. Using the formulas at the bottom of the Heat Loss Estimate Sheet calculate:
 - a) Kilowatt equivalent of the total heat loss
 - b) Yearly kilowatt hours
 - c) Estimated annual electrical power cost, based on the current rate. (Find the current rate by telephoning the electric utility company serving your area and ask for the rate.)

60 Design Temperature Difference

Construction	Area Sq. Ft.	BTU/Sq Ft/Hr	BTU/Hr.
<u>Windows & Doors</u>			
Single glass (single doors)	320	90	
Double glass		60	
<u>Glass Doors</u>			
Single glass		150	
Double Glass	160	120	
<u>Walls</u>			
Wood frame, no insulation		17	
Wood frame, $\frac{1}{2}$ " insulation		7	
Wood frame, 2" insulation/	1,364	5	
Concrete, no insulation, above grade		30	
Concrete, 2" insulation, above grade		6	
Concrete, no insulation, below grade	668	4	

FORMULAS
Level III

Industrial, Electrical and
Hydraulics

B. Problems without Solutions

6. continued

<u>Ceilings</u>			
1" rigid (open beam)		10	
2" rigid (open beam)		8	
4" insulation		5	
6" insulation	1,830	3	
<u>Floors</u>			
No insulation		17	
2" insulation (foil with 2" air gap)	708	5	
Slab (BTU/foot of peri- meter above grade)		50	
Square footage of living area <u>2816</u> ;		Total	
BTU/HR/SQ.FT. _____ ;		BTU/Hr	

$$\left(\frac{\quad}{3413 \text{ BTU/KWH}} \right) \text{BTU/Hr} = (\quad) \text{KW}, (\quad) \text{KW} \times 1312,4 = (\quad) \text{KWH/Yr.}$$

$$(\quad) \text{KWH/Yr.} \times \text{Electric power cost/KWH} = (\quad) \text{Est. annual cost.}$$

FORMULAS
Level III

Industrial, Electric and
Hydraulics

Complete solutions to B problems

$$\begin{aligned}
 3. \quad \text{Resistance} &= \frac{\text{voltage}}{\text{Current}} \\
 &= \frac{.050 \text{ Volts}}{25 \text{ amperes}} \\
 &= .002 \text{ ohms}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \text{From ohm's law } IR &= E \\
 \text{The problem state maximum allowable } IR &= (.01) (E) \\
 &= (100A) (R) = (.01) (200V) \\
 R &= \frac{(.01) (200V)}{100A} \\
 R &= .02\Omega
 \end{aligned}$$

$$\begin{aligned}
 \text{By proportion } \frac{.02\Omega}{10\Omega} &= \frac{100 \text{ ft}}{x \text{ circular mills-ft}} \\
 (.02\Omega) (x \text{ cm-ft}) &= (100 \text{ ft}) (10\Omega) \\
 x \text{ cm-ft} &= \frac{(100 \text{ ft}) (10\Omega)}{(.02\Omega)} \\
 x \text{ cm-ft} &= 50,000 \text{ ft} \\
 x \text{ circular mills} &= 50,000
 \end{aligned}$$

Complete solutions to B problems

5.

Room & Size		LR-DR		KITCH-NOOK		UTILITY		BR #1		BR# 2		BR# 3	
FACTOR*		SQ		SQ		SQ		SQ		SQ		SQ	
		FT	BTU	FT	BTU	FT	BTU	FT	BTU	FT	BTU	FT	BTU
WINDOWS	90	90	8550	15	1350	9	810	33	2970	18	1620	18	1620
DOORS, wood	80	20	1600			40	3200						
SLD- Glass	150			40	6000								
WALLS	4	373	1492	105	420	59	236	223	892	78	312	78	312
PARTITIONS	5			24	120	140	700						
CEYLINGS	3	592	1776	263	789	100	300	256	768	192	576	180	540
FLOORS	5	592	2960	263	1315	100	500	256	1280	192	960	180	900
TOTAL BTU LOSS		16378		9994		5746		5910		3468		3372	
INSTALLED BTU		17065		10239		5972		5972		4266		3413	
INSTALLED WATTS		5000		3000		2000		1750		1250		1000	

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Regulations of the City of

GLASS 221 SQ. FT. X 90 FACTOR* = 19890 BTU PER HOUR * Heat loss factors shall comply with the National Environmental Systems Contractors Association, Manual J, Third Edition, 1967, "Load-Calculation." Said manual shall be filed with the City Auditor, and the Bureau of Buildings.

 SQ. FT. X FACTOR* = BTU PER HOUR

DOORS 60 SQ. FT. X 80 FACTOR* = 4800 BTU PER HOUR

80 SQ. FT. X 150 FACTOR* = 12000 BTU PER HOUR

NET 1123 SQ. FT X 4 FACTOR* = 4492 BTU PER HOUR

WALL 164 SQ. FT. X 5 FACTOR* = 820 BTU PER HOUR

CEYLINGS 2092 SQ. FT. X 3 FACTOR* = 6276 BTU PER HOUR

FLOORS 2092 SQ. FT. X 5 FACTOR* = 10460 BTU PER HOUR

TOTAL BUILDING HEAT LOSS = 58,738 BTU P/H + 10% 64,612 BTU

FORMULAS
Level III

Industrial, Electrical and
Hydraulics

Complete solutions to problems B

5.

Rooms & Size		FAM-RM		BATH#1		BATH#2		HALL					
	FACTOR*	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU	SQ FT	BTU
WINDOWS	90	18	1620	6	540	9	810	0	0				
DOORS, WOOD	80												
SLD-GLASS	150	40	6000										
WALLS	4	102	408	50	200	55	220	0	0				
PARTITIONS													
CEILINGS	3	204	792	60	180	80	240	105	315				
FLOORS	5	264	1320	60	300	80	400	105	525				
TOTAL BTU LOSS		10140		1220		1670		840					
INSTALLED BTU		10239		2560		2560		1706					
INSTALLED WATTS		3000		750(min)		750(min)		500(min)					

I agree to install according to the Heating and Ventilating
Regulations if the City of

Installed watts figured = 18.75KW

BTU P/H + 10% loss = 18.931KW > Installed watts

Change BR#1 from 1750W to 2000W increasing installed watts figured to
20.75KW < BTU P/H + 10% loss.

FORMULAS
Level III

Industrial, Electrical, and
Hydraulics

Complete solutions to B problems

6. 60° Design Temperature Difference

Construction	Area Sq.Ft.	BTU/Sq Ft/Hr	BTU/Hr.
<u>Windows & Doors</u>			
Single glass (single doors)	320	90	28,800
Double glass		60	
<u>Glass Doors</u>			
Single glass		150	
Double glass	160	120	19,200
<u>Walls</u>			
Wood frame, no insulation		17	
Wood frame, 1 1/2" insulation		7	
Wood frame, 2" insulation	1364	5	6,820
Concrete, no insulation, above grade		30	
Concrete, 2" insulation, above grade		6	
Concrete, no insulation, below grade	668	4	2,672
<u>Ceilings</u>			
1" rigid (open beam)		10	
2" rigid (open beam)		8	
4" insulation		5	
6" insulation	1830	3	5,490
<u>Floors</u>			
No insulation		17	
2" insulation (foil with 2" air gap)	708	5	3,540
Slab(BTU/foot of peri- meter above grade		50	
Square footage of living area 2816; Total BTU/HR/SQ.FT. 23.22 $\approx$ 24 ; BTU/Hr 66,522			

$$\frac{(66,522) \text{ BTU/Hr.}}{3413 \text{ BTU/KWH}} = (19.49) \text{ KW. } (19.49) \text{ KW} \times 1312.4 = (25,580) \text{ KWH/Yr.}$$

$$(25,580) \text{ KWH/Yr.} \times \text{Electric power cost/KWH} = (\$328.00) \text{ Est. annual cost.}$$

B. Problems without Solutions

1. There were 32 jacks used in lifting the center span of the Fremont Bridge. If the total load is 12,000,000 lbs, then each jack will lift about 375,000 lbs. The lifting rods were 4 in diameter steel rods with an area of $(0.785)(16) = 12.5$ sq in. How far will the 175 ft rod stretch with this load applied?

Note: $E = \frac{(P)(L)}{(A)(\Delta L)}$ where E = modulus of elasticity $\left(30,000,000 \frac{\text{lbs}}{\text{sq in}}\right)$

P = load or tensile force

L = length of material

A = Cross-sectional area of the rods

ΔL = change in length

B. Problems without Solutions (continued)

2. A frequent method of constructing bridge piers in water is to use a cofferdam. This is analogous to pushing an empty glass down into the water in a kitchen sink. Water exerts a buoyant force tending to float the water glass or cofferdam. The force depends on the displaced water (ignoring any weight of the cofferdam.)

After the cofferdam is constructed, it is desirable to dewater the cofferdam, so bridge foundations may be constructed "in the dry." To seal the bottom of the cofferdam against water infiltration and also to resist the buoyant "floating" force, a concrete seal or plug is placed in the bottom of the cofferdam. This seal of concrete hardens, the water is pumped out.

On this bridge, the cofferdam is 25 ft long and 15 ft thick. If the buoyant force from the water is equal to the weight of the displaced water, how much concrete in ft of depth, is needed to counter balance the buoyancy.

Note: Concrete weighs 150 lbs/cu ft and water weighs 62.4 lbs/cu ft

Complete Solutions to B Problems

1. Since $E = \frac{(P)(L)}{(A)(\Delta L)}$

$$L = \frac{(P)(L)}{(A)(E)}$$

Substituting the values from the problem,

$$L = \frac{(375,000 \text{ lbs}) [(175 \text{ ft}) (12 \text{ in/ft})]}{[(3.14) (4/2 \text{ in})^2] [30,000,000 \text{ lbs/sq in}]}$$

$$L = \frac{(375,000 \text{ lbs}) (2,100 \text{ in})}{(12.57 \text{ sq in}) (30,000,000 \text{ lbs/sq in})}$$

$$L = 2.1 \text{ inches}$$

2. Since the buoyancy equals the weight of the water displaced, we must solve for the volume of water displaced.

$$\begin{aligned} \text{Volume} &= (\text{length})(\text{thickness})(\text{height}) \\ &= (25 \text{ ft})(15 \text{ ft})(30 \text{ ft}) \\ &= 11,250 \text{ cu ft} \end{aligned}$$

Therefore, the weight of water displaced is
 $(11,250 \text{ cu ft})(62.4 \text{ lbs/cu ft}) = 702,000 \text{ lbs}$

If the concrete weighs 150 lbs/cu ft, you need

$$\frac{702,000 \text{ lbs}}{150 \text{ lbs/cu ft}} = 4,680 \text{ cu ft}$$

To find the depth of the concrete, divide this by the cross-sectional area of the cofferdam

$$\frac{4,680 \text{ cu ft}}{(15 \text{ ft})(25 \text{ ft})} = \frac{4,680 \text{ cu ft}}{375 \text{ sq ft}} = 12.5 \text{ ft}$$

FORMULAS
Level II

Forestry

A. Problems with solutions.

1. Timber companies are engaged in the production of particle board. One of their concerns is the density of the product which is calculated by means of the following formula:

(Note: To convert from gm/cu in to lb/cu ft, multiply by 3.81)

$$D = \frac{(3.81)(W)}{(w)(l)(t)} \quad \text{where; } D = \text{density of particle board (lbs/cu ft)}$$

W = weight of strip tested (gms)

l = length of strip tested (in)

w = width of strip tested (in)

t = thickness of strip tested (in)

Determine the density of the particle board for the following test data:

a) W = 257.8g, t = .501 in, l = 14 in, w = 3 in.

b) W = 553.6g, t = .760 in, l = 20 in, w = 3 in.

c) W = 153.2g, t = .375 in, l = 11 in, w = 3 in.

Solution:

$$a) \quad D = \frac{3.81W}{w l t} = \frac{3.81 (257.8 \text{ gms})}{3 \text{ in } (14 \text{ in}) (.501 \text{ in})} = 46.7 \text{ lbs/cu.ft.}$$

$$b) \quad D = \frac{3.81W}{w l t} = \frac{3.81 (553.6 \text{ gms})}{3 \text{ in } (20 \text{ in}) (.761 \text{ in})} = 46.2 \text{ lbs/cu.ft.}$$

$$c) \quad \frac{3.81W}{w l t} = \frac{3.81 (153.2 \text{ gms})}{3 \text{ in } (11 \text{ in}) (.375 \text{ in})} = 47.2 \text{ lbs/cu.ft.}$$

A. Problems with Solutions (continued)

2. In their quality control of particle board, Timber Engineering is interested in the strength of the product. One measure of strength is what is called the modulus of elasticity M_E which is determined by the following formula:

$$M_E = \frac{(L^3)X}{400W(T^3)} \quad \text{where } M_E = \text{modulus of elasticity (1,000 lb/sq in)}$$

L = length of sample (in)

X = load applied to sample (1,000 lbs)

W = width of sample (in)

T = thickness of sample (in)

Determine M_E for each of the following sets of raw data:

- a) $L = 9$ in, $T = 0.378$ in, $W = 3$ in, $X = 46$ (1000 lbs)
b) $L = 12$ in, $T = 0.503$ in, $W = 3$ in, $X = 42$ (1000 lbs)
c) $L = 15$ in, $T = 0.627$ in, $W = 3$ in, $X = 30$ (1000 lbs)

Solution:

$$\begin{aligned} \text{a) } M_E &= \frac{L^3 X}{(400 \text{ in})W(T^3)} = \frac{(9 \text{ in})^3 (46 \text{ 1000-lbs})}{(400 \text{ in}) (3 \text{ in}) (0.378 \text{ in})^3} \\ &= \frac{33,534 \text{ 1000-lbs}}{6.481 \text{ sq in}} \end{aligned}$$

$$= 517.4 \text{ 1000-lbs/sq in}$$

$$\begin{aligned} \text{b) } M_E &= \frac{L^3 X}{(400 \text{ in})W(T^3)} = \frac{(12 \text{ in})^3 (42 \text{ 1000-lbs})}{(400 \text{ in}) (3 \text{ in}) (0.503 \text{ in})^3} \\ &= \frac{72,576 \text{ 1000-lbs}}{152.7 \text{ sq in}} \end{aligned}$$

$$= 475.2 \text{ 1000-lb/sq in}$$

$$\begin{aligned} \text{c) } M_E &= \frac{L^3 X}{(400 \text{ in})W(T^3)} = \frac{(15 \text{ in})^3 (30 \text{ 1000-lbs})}{(400 \text{ in}) (3 \text{ in}) (0.627 \text{ in})^3} \\ &= \frac{101,250 \text{ 1000-lbs}}{295.8 \text{ sq in}} \end{aligned}$$

$$= 342.3 \text{ 1000-lbs/sq in}$$

A. Problems with Solutions (continued)

3. The forest service uses a formula for determining the volume of trees.

$$V = 6.15248 + (0.5254)(D)(F) - 0.28833H + 0.0377(DF)^2 + (0.02182)(H)(DF)^2 - (0.07303)(D)(F)(H)$$

where V = volume (board ft)
 D = diameter breast height (in)
 H = total height of tree, stump to tip (ft)
 F = form class (ratio of diameter inside bark to diameter outside bark)

Determine the board foot volume of the following trees:

- a) $D = 20$ in, $H = 110$ ft, $F = 0.78$
b) $D = 33$ in, $H = 90$ ft, $F = 0.84$

Solution:

$$\begin{aligned} \text{a) } V &= 6.15248 + (0.5254)(20 \text{ in})(0.78) - (0.28833)(110) + \\ &\quad (0.0377) [(20)(0.78)]^2 + (0.02182)(110) [(20)(0.78)]^2 - \\ &\quad (0.07303)(20)(0.78)(110) \\ &= 6.15248 + 8.19624 - 31.7163 + 9.174672 + 584.112672 - \\ &\quad 125.31948 \\ &= 450.60 \text{ bd ft} \end{aligned}$$

$$\begin{aligned} \text{b) } V &= 6.15248 + (0.5254)(33)(0.84) - (0.28833)(90) + \\ &\quad (0.0377) [(33)(0.84)]^2 + (0.02182)(90) [(33)(0.84)]^2 - \\ &\quad (0.07303)(33)(0.84)(90) \\ &= 6.15248 + 14.564088 - 25.9497 + 28.9686 + 1,508.9807 - \\ &\quad 182.195244 \\ &= 1,350.52 \text{ bd ft} \end{aligned}$$

FORMULAS
Level II

Forestry

A. Problems with solutions, continued.

4. The diameter at breast height of a ponderosa pine can be determined from an aerial photograph by the following formula:

$$D = 3.7600 + .01348(H \times V) - (2.4459) (10^{-6}) (H \times V)^2 + (2.4382) \times (10^{-10}) (H \times V)^3$$

where d = diameter at breast height (in)

H = total height of tree (ft)

V = visible crown diameter from photograph

Determine D for the following ponderosa pines:

a) $V = 22$, $H = 108$

b) $V = 8$, $H = 54$

Solution:

a) $D = 3.7600 + 0.01348 (22 \times 108) - (2.4459) (10^{-6}) (108 \times 22)^2 + (2.4382) (10^{-10}) (108 \times 22)^3 = 3.7600 + 32.0248 - 13.808025 + 3.27045 = 25.25 \text{ in.}$

b) $D = 3.7600 + 0.01348 (8 \times 54) - (2.4459) (10^{-6}) (8 \times 54)^2 + (2.4382) (10^{-10}) (8 \times 54)^3 = 3.7600 + 5.82336 - 0.045646 + 0.019136 = 9.557 \text{ in.}$

B. Problems without solutions.

5. Although log cross sections rarely form true circles, they are normally presumed to be circular for purposes of computing cross sectional areas, called basal area. If the diameter is designated by D and is in inches then the basal area B is given by:

$$B \text{ (sq. in.)} = \frac{\pi D^2}{4} \text{ where } \pi = 3.1416$$

$$B \text{ (sq ft)} = B \text{ (sq in.)} / 144 \quad \frac{\text{sq. in.}}{\text{Sq. ft.}} = \frac{\pi D^2}{(4) (144)} = 0.00545D^2$$

Determine the basal area in both sq. in and sq. ft if the diameter of the log is:

- a) 16 in.
- b) 38 in.

B. Problems without solutions, continued.

6. The reduction in scale for inferior aspects in a log can be accomplished by the following standard rule:

$$D = \frac{(w) (t) (l)}{15}$$

where D = deduction in board ft.

l = length of defect in feet

w = width of defect in inches (add 1 inch for trim.)

t = Thickness of defect in inches, (add 1 inch for trim).

- a) A fire scar at the butt of a log penetrates 4 in. into the cylinder and has an average width of 10 in. If the length of the scar is 4 ft, what is the loss in bd-ft from this scar?
- b) Rot extends completely through the center of a 20 ft long log, having an average width of 7 inches and thickness of 8 inches. What is the loss in bd-ft?

B. Problems without solutions, continued.

7. In their quality control work in the production of particle board, Timber Engineering Co. is interested in the strength of the product. One measure of the strength is what is called the modulus of rupture M_R which is given by the following formula:

$$M_R = \frac{.5LU}{T^2}$$

where M_R = modulus of rupture (lb/in²)

L = length of test sample (in)

T = thickness of test sample (in)

U = ultimate load (lbs) ie., load to break test sample.

Determine M_R for each of the following samples:

- a) L = 15 in, T = .629 in, U = 149 lbs
- b) L = 12 in, T = .496 in, U = 66 lbs
- c) L = 18 in, T = .758 in, U = 210 lbs.

FORMULAS

Forestry

Level II

B. Problems without solutions, continued.

8. The scale diameter of each log above the butt log (up to a diameter 50% of scaling diameter of the butt log) is given by the formula:

$$D = \frac{L}{.9L + 1.1} + .5 \text{ where}$$

$$D = \frac{\text{scaling diameter at any point}}{\text{scaling diameter of butt log}} \quad \text{and}$$

$$L = \frac{\text{no 16ft logs between any point and merchantable top}}{\text{no 16ft logs between top of butt log and merchantable top}}$$

Determine D given

a) $L = 5/6 = .833$

b) $L = 5/8 = .625$

Complete solutions to 3 problems.

5. Solution:

$$a) \quad B = \frac{3.1416 (16 \text{ in})^2}{4} = 201 \text{ sq. in.}$$

$$B = .00345 (16 \text{ in})^2 = 1.395 \text{ sq. ft.}$$

$$b) \quad B = \frac{3.1416 (38 \text{ in})^2}{4} = 1134 \text{ sq. in.}$$

$$B = .00545 (38 \text{ in})^2 = 7.87 \text{ sq. ft.}$$

6. Solution:

$$a) \quad D = \frac{(4 + 1)(10 + 1)}{15} \cdot 4 = \frac{(5)(11)}{15} \cdot 4 = 14.67 \text{ Bd. ft.}$$

$$b) \quad D = \frac{(7 + 1)(8 + 1)}{15} \cdot 20 = \frac{(8)(9)}{15} \cdot 20 = 96 \text{ Bd. ft.}$$

7. Solution:

$$a) \quad M_R = \frac{(0.5)(15)(149)}{(0.629)^2} = \frac{1117.5}{0.396} = 2824 \text{ lb/sq in}$$

$$b) \quad M_R = \frac{(0.5)(12)(66)}{(0.496)^2} = \frac{396}{0.2460} = 1610 \text{ lb/sq in}$$

$$c) \quad M_R = \frac{(0.5)(18)(210)}{(0.758)^2} = \frac{1890}{0.5746} = 3289 \text{ lb/sq in}$$

FORMULAS

Level II

Forestry

Complete solutions to B problems, continued.

8. Solution:

$$a) D = \frac{.833}{.9(.833) + 1.1} + .5 = \frac{.833}{1.8497} + .5 =$$

$$= .45 + .5 = .95$$

$$b) D = \frac{.625}{.9(.625) + 1.1} + .5 = \frac{.625}{1.6625} + .5 = .38 + .5 = .88$$

A. Problems with Solutions

1. Two shims are to have a combined thickness of 0.070". One shim is to be 2.5 times as thick as the other. What are their respective thicknesses?

Solution:

x = thickness of thinnest shim

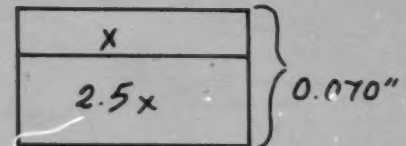
$2.5x$ = thickness of thickest shim

$$x + 2.5x = 0.070"$$

$$3.5x = 0.070"$$

$$x = 0.020"$$

$$2.5x = 0.050"$$



2. The formula for finding the volume of a frustum is:

$$V = \frac{1}{3} h (b + B + \sqrt{Bb})$$

B = area of large base, b = area of small base, and h = height.
A concrete base for a heavy machine has the form of a frustum of a rectangular pyramid. The top is 16' 4" x 4' 8" and the bottom is 32' 8" x 9' 4". The depth is 3' 8". What is the volume?

Solution:

$$h = 3' 8" = 3.67'$$

$$B = (32' 8") (9' 4")$$

$$= (32.67') (9.33') = 305 \text{ ft}^2$$

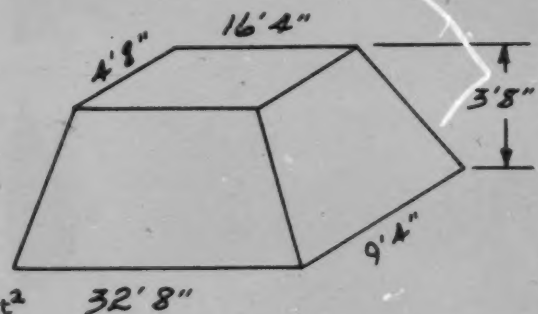
$$b = (4' 8") (16' 4")$$

$$= (4.67') (16.33') = 76.3 \text{ ft}^2$$

$$V = \left(\frac{1}{3}\right) (3.67') (305 \text{ sq ft} + 76.3 \text{ sq ft} + \sqrt{(305)(76.3)})$$

$$V = \left(\frac{1}{3}\right) (3.67') (381.3 \text{ sq ft} + 153 \text{ sq ft})$$

$$V = 653 \text{ ft}^3$$

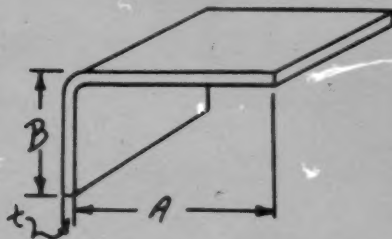


B. Problems without Solutions

3. To make a sharp bend in sheet metal, the required length is determined by the formula:

$$L = A + B + 1/2t$$

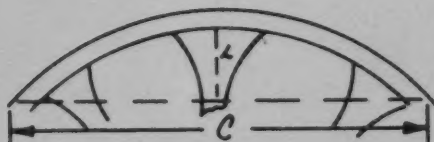
Find L when $A = 3 \frac{1}{2}"$, $B = 13.6"$, and $t = \frac{3}{8}"$



4. The length of a chord of a circle is given by the formula:

$$S = h(2r - h)$$

where S = length of chord; r = radius of the circle; and h = height of the circumference above the midpoint of the chord. Suppose a broken flywheel is to be replaced by a new flywheel of the same size. The largest piece of the broken wheel has its longest chord (S) equal to 16", and the height (h) of the circumference above the chord is 6". What is the radius of the flywheel?



B. Problems without Solutions (continued)

5. The volume of an elliptical tank is given by the formula:

$$V = (\text{width})(\text{length})(\text{height})(0.7854)$$

If an elliptical tank has a width of 5', a height of 3', and a length of 12', how many gallons of water will it hold?
(1 cu ft = 7.48 gal)

6. When the pressure of the water at the place of discharge is denoted by a gage, the rate of discharge may be determined by the formula: $V = (12.16)P$, where V = velocity in feet per second and P = pressure in pounds per square inch, as read from the gage. Compute the flow under pressure of 225 lbs/sq in.

7. The lifting capacity of a sling is reduced as the angle of its legs to the horizontal is reduced. When determining the length and size of the sling, the tension on each leg must be calculated; it is given by the formula: $T = \frac{W}{N} \times \frac{L}{V}$

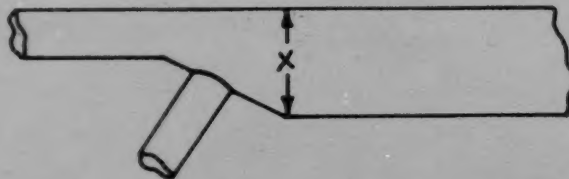
T = tension; W = vertical distance from the top of the load to the hook. Determine the tension of a single leg of a two legged sling being used to lift a load weighing 1800 pounds, if $L = 8'$ and $V = 6'$. By knowing the tension, we can then determine the size of wire, chain, or rope to use in making the sling.

8. On a certain chain driven line, a 6" sprocket drives a 10" sprocket. The speed of the line needs to be reduced 15%. If the 6" sprocket must remain in the line, what size sprocket should be substituted for the 10" sprocket?

B. Problems without Solutions (continued)

9. A millwright receives an $8\frac{1}{2}\%$ raise which increases his pay 35¢ per hour. How much does he receive an hour after his raise?

10. Given two joining pipes 12" and 8" in diameter, find the diameter x of the continuation which has the same capacity as the other two smaller pipes.



B. Problems without Solutions (continued)

11. A mixture of resin, water, and wax is used at a plywood mill to make a finish. The resin is delivered by truck as a 65% solution; that is 65% resin, 35% water. It is pumped into a 1,000 gallon tank where it becomes the finish by mixing it with wax. The wax is $1/2\%$ of the mixture. How many gallons of each substance, resin, water, and wax are in the tank?

12. A machinist has been given the job of milling "hex" heads on both ends of 62 shafts. A crane is required to lift the shafts into place. Each shaft requires 4 trips (2 up and 2 down) of the crane. He has a choice of two cranes; one which has a lifting speed of 5 ft per minute and another which has a lifting speed of 17 ft per minute. If the shafts need to be lifted 5' and he is paid \$5.26 per hour, how much money can be saved by using the faster crane?

SIMPLE EQUATIONS AND FORMULAS
Level 2

Industrial Mechanics

Complete Solutions to B Problems

3. $L = A + B + 1/2t$

$$L = 3 \frac{1}{2}'' + 13.6'' + (1/2)(3/8'')$$

$$L = 3.5'' + 13.6'' + (0.5)(0.375'')$$

$$L = 17.1'' + 0.1875''$$

$$L = 17.2875'' \approx 17 \frac{9}{32} \text{ in}$$

4. $S = h(2r - h)$ $h = 6''$ $S = 16''$

$$16'' = (6)(2r - 6)$$

$$16'' = 12r - 36''$$

$$52'' = 12r$$

$$r = \frac{52''}{12} = 4.33 \text{ in}$$

5. $V = (5')(12')(3')(0.7854) = 141.4 \text{ cu ft}$

$$\text{Volume in gallons} = (141.4 \text{ cu ft})(7.48 \text{ gal/cu ft}) = 1,058 \text{ gal}$$

6. $V = (12.16)P$

$$V = (12.16)(225)$$

$$V = 2,736 \text{ ft/sec}$$

7. $T = \left(\frac{W}{N}\right)\left(\frac{L}{V}\right) = \left(\frac{1,800 \text{ lbs}}{2}\right)\left(\frac{8 \text{ ft}}{6 \text{ ft}}\right)$

$$= \frac{14,400 \text{ lbs}}{12} = 1,200 \text{ lbs}$$

8. Reduce diameter of 10" sprocket by 15%

$$10'' - (0.15)(10'') = 10'' - 1.5''$$

$$= 8.5 \text{ in}$$

Complete Solutions to B Problems (continued)

9. x = original hourly wage

$$8\frac{1}{2}\%x = 0.085x = \text{increase in wage}$$

$$0.085x = \$0.35$$

$$x = \frac{\$0.35}{0.085} = \$4.12$$

$$\text{New hourly wage} = \$4.12 + (0.085)(\$4.12)$$

$$= \$4.12 + \$0.35$$

$$= \$4.47$$

10. The continuation must have a cross-sectional area equal to the total cross-sectional areas of the 8" and 12" pipes

$$\text{Total cross-sectional area} = \pi(4 \text{ in})^2 + \pi(6 \text{ in})^2 = 163.3 \text{ sq in}$$

$$\text{Cross-sectional area of large pipe} = \pi x^2$$

$$\pi x^2 = 163.3 \text{ sq in}$$

$$3.14x^2 = 163.3 \text{ sq in}$$

$$x^2 = \frac{163.3 \text{ sq in}}{3.14}$$

$$x^2 = 52.0 \text{ sq in}$$

$$x = \sqrt{52.0 \text{ sq in}}$$

$$x = 7.21 \text{ in}$$

$$\text{diameter} = 2x = 14.42 \text{ in}$$

11. $Wax = (1/2\%)(1,000 \text{ gal}) = (0.005)(1,000 \text{ gal}) = 5 \text{ gal}$

$$\text{Volume of resin and water} = 1,000 \text{ gal} - 5 \text{ gal} = 995 \text{ gal}$$

$$\text{Volume of resin} = (65\%)(995 \text{ gal}) = (0.65)(995 \text{ gal}) = 647 \text{ gal}$$

$$\text{Volume of water} = (35\%)(995 \text{ gal}) = (0.35)(995 \text{ gal}) = 348 \text{ gal}$$

Complete Solutions to B Problems (continued)

12. Need to find the difference between the lifting times for the two cranes.

Lifting time for slower crane (5 ft/min)

$$\text{Time for one trip} = \frac{5 \text{ ft}}{5 \text{ ft/min}} = 1 \text{ min}$$

$$\begin{aligned} \text{Total time for lifting} &= (4 \text{ trips/shaft}) (62 \text{ shafts}) (1 \text{ min/trip}) \\ &= 248 \text{ min} = 4.13 \text{ hrs} \end{aligned}$$

$$\text{Cost of lifting} = (4.13 \text{ hrs}) (\$5.26/\text{hr}) = \$21.74$$

Lifting time for faster crane (17 ft/min)

$$\text{Time for one trip} = \frac{5 \text{ ft}}{17 \text{ ft/min}} = 0.29 \text{ min}$$

$$\begin{aligned} \text{Total lifting time} &= (4 \text{ trips/shaft}) (62 \text{ shafts}) (0.29 \text{ min/trip}) \\ &= 71.92 \text{ min} = 1.20 \text{ hrs} \end{aligned}$$

$$\text{Cost of lifting} = (1.20 \text{ hrs}) (\$5.26/\text{hr}) = \$6.31$$

$$\text{Savings} = \$21.74 - \$6.31 = \$15.43$$

A. Problems with Solutions

1. What voltage is behind a current that flows through a motor at 20 amperes, producing 5 kilowatts of power?

$$\text{Use } KW = \frac{IE}{1000}$$

Solution:

Substitute the given numbers into the equation and solve for E

$$KW = \frac{IE}{1000}$$

$$5 = \frac{20E}{1000} \quad \text{multiply both sides by 1000}$$

$$5000 = 20E \quad \text{divide both sides by 20}$$

$$250 = E \quad \text{So there are 250 volts behind this current}$$

2. Determine the radius of curvature of a piece of headlamp taken from a wreck scene which has a distance from the screw to the leg of 4 inches and the distance of the screw movement is 0.12 inches using the formula $R = \frac{h^2}{2L} + \frac{L}{2}$ where R = radius of curvature, h = distance from leg to screw and L = distance the screw moves.

Solution:

Substitute the given values into the equation and solve for R

$$R = \frac{h^2}{2L} + \frac{L}{2}$$

$$R = \frac{(4 \text{ inches})^2}{2(0.12 \text{ in})} + \frac{0.12 \text{ inches}}{2}$$

$$= \frac{16 \text{ sq in}}{0.24 \text{ inches}} + 0.06 \text{ inches}$$

$$= 66.67 \text{ inches} + 0.06 \text{ inches} = 66.73 \text{ inches}$$

Therefore, the radius of curvature is 66.73 inches.

A. Problems with Solutions (continued)

3. A rock is thrown (nearly) vertically upward from the cornice of a tall building, leaving the thrower's hand with a speed of 48 ft/sec and just missing the cornice on the way down. Find (a) the position of the rock, 1 second and 4 seconds after leaving the thrower's hand; (b) the velocity when the rock is 20 ft above its starting point.

Solution:

There are two things to solve for. First, use the equation $S = V_0 t + \frac{1}{2} a t^2$, where V_0 is the velocity, t is the time, and $a = -32 \text{ ft/sec}^2$. Second, use the equation $V^2 = V_0^2 + 2 a s$ where s is the distance from the starting point, and $a = -32 \text{ ft/sec}^2$.

(a) For one second

$$S = V_0 t + \frac{1}{2} a t^2$$

$$S = (48 \text{ ft/sec})(1 \text{ sec}) + (0.5)(-32 \text{ ft/sec}^2)(1 \text{ sec})^2$$
$$S = 32 \text{ feet up in one second}$$

For 4 seconds

$$S = V_0 t + \frac{1}{2} a t^2$$

$$S = (48 \text{ ft/sec})(4 \text{ sec}) + (0.5)(-32 \text{ ft/sec}^2)(4 \text{ sec})^2$$
$$S = -64 \text{ feet or 64 down the building at four seconds}$$

(b) $V^2 = V_0^2 + 2 a s$

$$V^2 = (48 \text{ ft/sec})^2 + (2)(-32 \text{ ft/sec}^2)(20 \text{ ft})$$
$$V^2 = 2,304 \text{ ft}^2/\text{sec}^2 + (-1,280 \text{ ft}^2/\text{sec}^2)$$
$$V^2 = 1,024 \text{ ft}^2/\text{sec}^2$$
$$V = 32 \text{ ft/sec}$$

So the velocity of the rock when it is 20 feet above its starting point is 32 feet/sec.

EQUATIONS AND FORMULAS
Level 2

Fire and Police Science

B. Problems without Solutions

4. A generator supplies 220 amps rated at 110 volts each using 200 watts. What is the power load? How many amperes are used? Use $KW = \frac{IE}{1000}$
5. What current will flow through a toaster using 550 watts at 110 volts? Use $W = I E$
6. Delf Bryce, of the F.B.I. can drop a silver dollar from his shooting hand at the level of his forehead, and have his gun out and shooting as the dollar passes the muzzle of his gun. Find the time, t , between the release of the dollar and its being hit if it falls 2.6 ft. Use $S = \frac{1}{2} g t^2$ where S is the distance the coin falls and $g = 32 \text{ ft/sec}^2$

Fire and Police Science

7. What is the acceleration in feet per second² of a bullet in a 6-inch barrel if its muzzle speed is 850 ft per second? Use $V^2 = 2as$ where V is the velocity and s is the distance traveled.

8. The focal length of an $f/2.8$ camera lens is 8 cm. What is the diameter of the lens? Use $F.L. = N D$ where N is 2.8
9. Camera A, having an $f/8$ lens 2.5 cm in diameter, photographs an object using the correct exposure of $1/100$ sec. What exposure should camera B use in photographing the same object if it has an $f/4$ lens 5 cm in diameter? The required time of exposure increases with the square of the f /number. Use $t = k N^2$ where N is the denominator of the f number.

EQUATIONS AND FORMULAS
Level 2

Fire and Police Science

B. Problems without Solutions (continued)

10. In the investigation of a death it was found that the plaster on the ceiling had become loose and fallen 14.6 ft before striking the victim. What was the speed of the plaster on impact? (The speed of the plaster does not depend upon its size or weight) Use $V^2 = 2as$ where V is the speed and a is 32 ft per sec² and s is the distance the plaster fell.

EQUATIONS AND FORMULAS
Level 2

Fire and Police Science

Complete Solutions to B Problems

4. You first need to determine the power load in kilowatts (KW)

$$\begin{aligned} KW &= \frac{IE}{1,000} \quad I = 220 \quad E = 200 \\ &= \frac{(220)(200)}{1,000} = \frac{44,000}{1,000} = 44 \text{ kilowatts} \end{aligned}$$

Now use the same formula to determine the number of amps used.

$$\begin{aligned} KW &= \frac{IE}{1,000} \\ 44 &= \frac{I(110)}{1,000} \quad \text{multiply both sides by 1000} \\ 44,000 &= I(110) \quad \text{divide both sides by 110} \\ 400 &= I \quad \text{So 400 amperes are used} \end{aligned}$$

5. You need to substitute the given values into the equation and solve for I.

$$\begin{aligned} W &= IE \\ 550 &= I(110) \quad \text{divide both sides by 110} \\ 5 &= I \quad \text{So the current flow is 5 amperes} \end{aligned}$$

6. You need to substitute the given values into the equation, and then solve for t

$$\begin{aligned} S &= 1/2 g t^2 \\ 2.6 \text{ ft} &= 1/2(32 \text{ ft/sec}^2) t^2 \quad \text{multiply by 1/2} \\ 2.6 \text{ ft} &= (16 \text{ ft/sec}^2) t^2 \quad \text{divide both sides by 16 ft/sec} \\ 0.1625 \text{ sec}^2 &= t^2 \\ 0.403 &= t \end{aligned}$$

So the time between the dollar's release and being hit is about 4/10 of a second.

Complete Solutions to B Problems (continued)

7. You need to substitute the given values into the equation and then solve for a.

$$v^2 = 2 a s$$

$$(850 \text{ ft/sec})^2 = 2(a) (6 \text{ inches}) \quad \text{change inches to feet}$$

$$(850 \text{ ft/sec})^2 = 2(a) (0.5 \text{ ft}) \quad \text{multiply}$$

$$(850 \text{ ft/sec})^2 = (a) (1 \text{ ft})$$

$$\frac{722,500 \text{ ft}^2/\text{sec}^2}{1 \text{ ft}} = a$$

$$722,500 \text{ ft/sec}^2 = a \quad \text{The acceleration is } 722,500 \text{ ft/sec}^2$$

8. You need to substitute the given values into the equation and solve for D

$$F.L. = N D$$

$$8 \text{ cm} = (2.8) (D) \quad \text{divide both sides by 2.8}$$

$$2.857 \text{ cm} = D \quad \text{The diameter of the lens is 2.86 cm}$$

9. You should first determine the constant K using the data from camera A. Then, use the same equation with your calculated K value to find the exposure for camera B.

$$t = K N^2$$

$$1/100 \text{ sec} = K (8)^2$$

$$\frac{1}{6,400} = K$$

$$t = K N^2$$

$$t = \left(\frac{1}{6,400} \right) (4)^2$$

$$t = \left(\frac{1}{6,400} \right) (16)$$

$$t = \frac{16}{6,400} = \frac{1}{400}$$

For camera B, we need an exposure time of 1/400 second.

Complete Solutions to B Problems (continued)

10. You need to substitute the given values into the equation, and then solve for V.

$$v^2 = 2 a s$$

$$v^2 = 2 (32 \text{ ft/sec}^2) (14.6 \text{ ft})$$

$$v^2 = 934 \text{ ft}^2/\text{sec}^2$$

$$v = 30.56 \text{ ft/sec}$$

The speed of the plaster was 30.6 feet per second on impact.

EQUIVALENTS

1 gallon water (sewage) = 8.34 pounds
7.48 gallons water (sewage) = 1 cubic foot water
1 cubic foot water (sewage) = $8.34 \times 7.48 = 62.4$ pounds
1 inch = 2.54 centimeters = 0.0833 feet
1 cubic foot = 1728 cubic inches
1 pound = 453.6 grams
1 cubic centimeter = 1 gram (water) = 1 milliliter
1 million gallons per day (MGD) = 694 GPM = 1.547 CFS
1 CFS (cubic foot per second) = 646,317 gallons/day = 449 gpm
1 liter = 1000 milliliters
1 milligram/liter (mg/l) = 1 part per million (ppm) = 8.34 pounds per
million gallons
1 foot head (water) = 0.433 psi (pounds per square inch)
1 pound per square inch (psi) = 2.31 ft of water
1 meter = 100 centimeters = 1000 millimeters = 3.281 feet
1 milliliter = 20 drops (approximately)
1 acre = 43,560 square feet
1 acre foot = 326,000 gallons
1 million gallons = 3.07 acre feet
1 horsepower = 0.746 kilowatts = 550 foot pounds/second = 33,000 ft lbs/minute
1 mile = 5,280 feet = 1,760 yards
1 gallon = 231 cubic inches
a square mile = 640 acres
1 cubic foot/sec = 0.99173 acre inches per hour
 $\pi = 3.1416$

ABBREVIATIONS OF UNITS AND MEASUREMENTS

in = inch

ft = foot

yd = yard

sec = second

hr = hour

mgd = million gallons per day

gal = gallon

lb or # = pound

mg/l = milligrams per liter

ppm = parts per million

in = square inches

in = cubic inches

ft = square feet

ft = cubic feet

CFS = cubic feet per second

FPS = feet per second

gpm = gallons per minute

HP = horsepower

psi = pounds per square inch

gph = gallons per hour

Q = flow (mgd, gpm, gph, cfs, etc.)

EQUATIONS
Level 1

Wastewater Tech

A. Problems with Solutions

1. Convert 75° F to degrees Celcius.
Convert 20° C to Fahrenheit.

Solution:

$$F^{\circ} = (9/5)C^{\circ} + 32$$

$$C^{\circ} = (5/9)(F^{\circ} - 32)$$

75° F

20° C

$$C^{\circ} = (5/9)(75^{\circ} - 32^{\circ})$$
$$= 23.888^{\circ}$$

$$F^{\circ} = (9/5)(20^{\circ}) + 32^{\circ}$$
$$= 68$$

2. A pump set at a rating of 80 gallons per minute is pumping sludge through a 6-inch line. What is the velocity of the sludge traveling through the line?

Solution:

$$Q = AV$$

Q = rate of flow in cfs

A = cross-sectional area in square feet

V = velocity of the flow in feet per second

$$V = \frac{Q}{A}$$

$$1 \text{ cfs} = 448.8 \text{ gpm} \quad \text{or} \quad 1 \text{ gpm} = 0.00223 \text{ cfs}$$

$$Q = (80 \text{ gpm})(0.00223 \text{ cfs/gpm}) = 0.1784 \text{ cfs}$$

$$A = (d^2)(0.7854) = (0.5 \text{ ft})(0.5 \text{ ft})(0.7854) = 0.19635 \text{ sq ft}$$

$$V = \frac{0.1784 \text{ cfs}}{0.19635 \text{ sq ft}} = 0.90858 \text{ ft/sec}$$

3. What is the velocity of flow in a rectangular grit channel which is 3 feet wide, has a depth of flow of one foot when the flow meter registers 2.5 MGD.

Solution:

$$Q = AV$$

where

Q = the rate of flow in cfs

A = the cross sectional area in square feet

V = the velocity of flow in feet per second

$$V = \frac{Q}{A}$$

$$1 \text{ MGD} = 1.547 \text{ cfs}$$

A. Problems with Solutions (continued)

3. Solution (continued)

To use the formula, Q must be in cfs units. So, change 2.5 MGD to cfs.

$$Q = (2.5 \text{ MGD}) (1.547 \text{ cfs/MGD}) = 3.8675 \text{ cfs}$$

$$A = (3 \text{ ft}) (1 \text{ ft}) = 3 \text{ sq ft}$$

$$V = \frac{Q}{A} = \frac{3.87 \text{ cfs}}{3 \text{ sq ft}} = 1.29 \text{ ft/sec}$$

Round off to two significant digits, $V = 1.3 \text{ ft/sec}$

B. Problems without Solutions

4. Three hundred ml BOD bottles were used in setting up a standard 5-day BOD test. The samples were as follows:

<u>Sample</u>	<u>Volume</u>	<u>D.O.</u>
Blank	(only H ₂ O)	7.8 ppm
Raw	10 ml	7.9 ppm
Primary	10 ml	7.6 ppm
Final	40 ml	7.2 ppm

After the 5-day incubation period, the D.O. tested:

<u>Sample</u>	<u>D.O.</u>
Blank	7.8 mg/l
Raw	2.0 mg/l
Primary	2.5 mg/l
Final	2.7 ppm

- (a) Calculate the BOD of the samples.
(b) What is the percent reduction between primary and final?
(c) What is the plant efficiency?

5. You want to find out just what your plant is doing to the stream into which your outfall flows. To do this, you go slightly upstream from the treatment plant and take a sample. This upstream sample shows a BOD of 4 ppm, and you know from DEO that a sample from just below the plant has a BOD of 7 mg/l. If the stream is flowing at 25.4 cfs, (a) how many pounds of BOD have been added to the stream if the effluent flow is 6 MGD? (b) what is the concentration of BOD in the effluent?

6. A large plant has seven sludge beds each measuring 55 feet by 100 feet. What is the total surface area of the beds expressed in both square feet and acres?
7. A large laboratory area is to be retiled using 18 inch by 18 inch tile. If the size of the room is 12 feet by 30 feet, how many tile will be needed?

B. Problems without Solutions (continued)

8. If the velocity in a 12-inch pipe flowing full is 4 fps, what is the rate of flow in cfs?

9. Reproduced below is a partial BOD record for a day in July. The volume of the bottles when capped is 300 ml.

	influent			effluent		
initial d.o.	8.9	8.9	8.9	8.9	8.9	8.9
final d.o.	4.9	3.4	1.9	5.7	4.9	3.2
depletion						
ml sample added	4	6	8	4	6	8
dilution factor						
BOD, ppm						

- (a) What are the depletion, dilution factor, and BOD values for this record?
- (b) What is the percentage BOD reduction for the primary sedimentation unit sampled?
- (c) With an average flow of 3.0 MGD for the day, what was the pounds BOD settled out in the primary sedimentation unit?

EQUATIONS
Level 1

Wastewater Tech

B. Problems without Solutions (continued)

9.

10. The digester gas is kept at a constant pressure equal to an 8" water column. What is the pressure in psi?

11. What is the velocity (ft/sec) of sludge in the 6" discharge pipe of a plunger pump with a discharge rate of 545 gals/0.1 hr?

B. Problems without Solutions (continued)

12.

(a) The centrifugal recirculation pump moves digesting sludge at the rate of 450 gpm. What is the velocity (ft/sec) in its 4" discharge line?

(b) After some distance, the 4" pipe gives way to a 6" pipe via an eccentric coupling. What is the velocity in the 6" pipe?

13. The primary sedimentation tank 96' by 16' has a 12" drain pipe situated at the deepest (12') end. The water level of the tank needs to be dropped 1' to check for corrosion on the outside surfaces of the effluent troughs. Once the tank is isolated, how long will it take to lower the water level the desired distance?

EQUATIONS
Level 1

Wastewater Tech

Complete Solutions to B Problems

4.
$$\text{BOD} = \frac{(\text{initial DO} - \text{final DO}) (\text{bottle volume})}{\text{sample volume before dilution}}$$

(a)
$$\text{BOD (raw)} = \frac{(7.9 \text{ ppm} - 2.0 \text{ ppm}) (300 \text{ mls})}{10 \text{ mls}} = 177 \text{ ppm}$$

$$\text{BOD (primary)} = \frac{(7.6 \text{ ppm} - 2.5 \text{ ppm}) (300 \text{ mls})}{10 \text{ mls}} = 153 \text{ ppm}$$

$$\text{BOD (final)} = \frac{(7.2 \text{ ppm} - 2.7 \text{ ppm}) (300 \text{ mls})}{40 \text{ mls}} = 34 \text{ ppm}$$

(b)
$$\frac{(153 - 33.75)}{153} = 0.78 = 78\%$$

(c)
$$\frac{(177 - 33.75)}{177} = 0.81 = 81\%$$

5.
$$\text{BOD pounds} = (\text{MGD}) (8.345 \text{ lbs/gal}) (\text{ppm})$$

$$(\text{cfs}) (0.65) = \text{MGD}$$

$$\text{Flow of stream} = (25.4 \text{ cfs}) (0.65) = 16.42 \text{ MGD}$$

Pounds BOD carried by stream:
$$(16.42 \text{ MGD}) (8.345 \text{ lbs/gal}) (4 \text{ ppm}) = 547.94 \text{ lbs}$$

$$\text{Flow of Stream below outfall} = 6 \text{ MGD} + 16.42 \text{ MGD} = 22.42 \text{ MGD}$$

Pounds BOD carried below the plant:
$$(22.42 \text{ MGD}) (8.345 \text{ lbs/gal}) (7 \text{ ppm}) = 1309.37 \text{ lbs}$$

$$\text{Pounds BOD added to stream} = 1309.37 \text{ lbs} - 547.94 \text{ lbs} = 761.37 \text{ lbs}$$

BOD concentration in the effluence:
$$761.37 \text{ lbs} = (6 \text{ MGD}) (8.345 \text{ lbs/gal}) (\text{ppm})$$

$$\text{ppm} = \frac{761.37 \text{ lbs}}{(6 \text{ MGD}) (8.345 \text{ lbs/gal})}$$

$$= 15.21 \text{ ppm}$$

6.
$$\text{Area} = (\text{length}) (\text{width})$$

$$= (100 \text{ ft}) (55 \text{ ft}) = 5,500 \text{ sq ft}$$

$$\text{Converting: } (5,500 \text{ sq ft}) (1 \text{ acre}/43,560 \text{ sq ft}) = 0.13 \text{ acres}$$

Complete Solutions to B Problems (continued)

7. There are several approaches to this problem:

- (a) Convert every measurement to inches.
- (b) Convert all measurements to feet.
- (c) Note that the size of the room is in exact multiples of tile size.

Most people would probably go with the conversion to feet.
Thus, the tiles each cover (1.5 sq ft) = 2.25 sq ft/tile and
the room is 12' by 30' or 360 sq ft making 360 sq ft/2.25 sq ft/tile
= 160 tile.

8. The standard flow formula is $Q = AV$ where,
 Q = the rate of flow in cubic feet per second
 A = the cross-sectional area perpendicular to the direction
of flow, expressed in square feet.
 V = the velocity of flow in feet per second.

$$\begin{aligned} Q &= AV \\ A &= \pi d^2/4 = \pi/4 \text{ square feet} \\ V &= 4 \text{ fps} \\ Q &= (\pi/4)(4) = \pi = 3.14159 \dots \text{cubic feet per second} \end{aligned}$$

9. (a) Depletion = initial d.o. - final d.o.
Dilution factor = ml bottle/ml sample
BOD, ppm = (depletion)(dilution factor)
Depletion 4.0, 5.5, 7.0, 3.2, 4.0, 5.7
Dilution factor 300/4 = 75, 300/6 = 50, 300/8 = 37.5
BOD, ppm (4.0)(75) = 300, (5.5)(50) = 275, (7.0)(37.5) = 262,
(3.2)(75) = 240, (4.0)(50) = 200, (5.7)(37.5) = 214

$$(b) \text{ BOD reduction, \%} = \left(\frac{\text{influent} - \text{effluent}}{\text{influent}} \right) (100)$$

$$\text{Influent average} = (300 + 275 + 262)/3 = 279$$

$$\text{Effluent average} = (240 + 200 + 214)/3 = 218$$

$$\left(\frac{279 - 218}{279} \right) (100) = 22\%$$

$$\text{BOD, lbs} = (\text{BOD, ppm})(8.34)(\text{flow, MGD})$$

(c) Influent BOD, ppm - effluent BOD, ppm = BOD removed

$$279 \text{ ppm} - 218 \text{ ppm} = 61 \text{ ppm}$$

$$(61 \text{ ppm})(8.34)(3.0 \text{ MGD}) = 1,526 \text{ lbs removed}$$

Complete Solutions to B Problems (continued)

10. A cubic foot of water weighs 62.4 pounds and exerts a pressure of 62.4 lbs/sq ft

$$\frac{62.4 \text{ lbs/sq ft}}{144 \text{ sq in/sq ft}} = 0.43 \text{ lbs/sq in} \quad \text{pressure equal to a 12" column}$$

$$\left(\frac{8''}{12''}\right) (0.43 \text{ lbs/sq in}) = 0.29 \text{ lbs/sq in}$$

11. $Q = AV$, therefore, $V = \frac{Q}{A}$

1st: convert 545 gals/0.1 hr to cu ft/sec

$$\left(\frac{545 \text{ gals}}{0.1 \text{ hr}}\right) \left(\frac{1 \text{ hr}}{60 \text{ min}}\right) \left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = 1.5 \text{ gals/sec}$$

$$\frac{1.5 \text{ gals/sec}}{7.5 \text{ gals/cu ft}} = 0.2 \text{ cu ft/sec}$$

2nd: cross-sectional area of pipe calculated

$$\text{Area} = (0.785) (0.5 \text{ ft}) (0.5 \text{ ft}) = 0.2 \text{ sq ft}$$

3rd: substitute values into equation above

$$V = \frac{0.2 \text{ cu ft/sec}}{0.2 \text{ sq ft}} = 1 \text{ ft/sec}$$

12. (a) $Q = AV$, therefore, $V = \frac{Q}{A}$

$$Q = \frac{450 \text{ gals/min}}{(7.5 \text{ gals/cu ft}) (60 \text{ sec/min})} = 1 \text{ cu ft/sec}$$

$$4 \text{ in} = 1/3 \text{ ft}$$

$$A = (0.785) (0.333 \text{ ft}) (0.333 \text{ ft}) = 0.09 \text{ sq ft}$$

$$V = \frac{1.0 \text{ cu ft/sec}}{0.09 \text{ sq ft}} = 11.1 \text{ ft/sec}$$

- (b) Q in 4" pipe equals Q in 6" pipe

$$Q = 1 \text{ cu ft/sec}$$

$$A = (0.785) (0.5 \text{ ft}) (0.5 \text{ ft}) = 0.2 \text{ sq ft}$$

$$V = \frac{1.0 \text{ cu ft/sec}}{0.2 \text{ sq ft}} = 5 \text{ ft/sec}$$

Compare to the velocity in the 4" pipe!

Complete Solutions to 8 Problems (continued)

13. The velocity of a unit mass (or volume) of water in the drain pipe will be the same as that achieved by a unit mass (or volume) falling from rest from the height of the water surface above the drain pipe opening. Although the surface level falls as the tank is drained, accompanied by a gradually decreasing velocity in the pipe, the small difference between initial and final height and the small decrease in velocity may be ignored for all practical purposes.

$$\text{P.E.} = \text{K.E. gives } dqh = \frac{1}{2}dv^2; \text{ rearranged: } v = \sqrt{2gh}$$

$$(2)(32 \text{ ft/sec})(12 \text{ ft}) = 27.7 \text{ ft/sec}$$

$$A = (0.785)(1 \text{ ft})^2 = 0.785 \text{ sq ft}$$

$$Q = (0.785 \text{ sq ft})(27.7 \text{ ft/sec}) = 21.7 \text{ cu ft/sec}$$

$$\text{Volume drained} = (96 \text{ ft})(16 \text{ ft})(1 \text{ ft}) = 1,536 \text{ cu ft}$$

$$\text{Time required} = \frac{1,536 \text{ cu ft}}{21.7 \text{ cu ft/sec}} = 71 \text{ sec}$$

A. Problems with Solutions

1. Most people tend to forget how to convert temperatures. The following technique is retained by most food processing technicians. To whatever temperature is given add 40° and multiply the sum times the appropriate fraction ($5/9$ going from Fahrenheit to Centigrade and $9/5$ going from Centigrade to Fahrenheit) and subtract 40° from this product.

Example: $F^{\circ} \rightarrow C^{\circ}$
 $212^{\circ} F + 40^{\circ} = (252^{\circ})(5/9) = 140^{\circ} - 40^{\circ} = 100^{\circ} C$

- Problem: (a) Convert $76^{\circ} F$ to Centigrade
(b) Convert -15 Centigrade to Fahrenheit

Solution:

(a) $76^{\circ} F + 40^{\circ} = (116^{\circ})(5/9) = 64.44^{\circ} - 40^{\circ} = 24.44^{\circ} C$

(b) $-15^{\circ} C + 40 = (25^{\circ})(9/5) = 45^{\circ} - 40^{\circ} = 5^{\circ} F$

2. If the SO_2 strength is lower than that desired, it will be necessary to add more SO_2 . The amount to add can be calculated roughly from the following formula:
(Percent SO_2 desired - percent SO_2 found) multiplied by the total gallons of brine and by 8.4, the weight of a gallon of brine, all divided by 100.

$$\frac{(\text{Percent difference}) \times (\text{gallons}) \times (8.4)}{100} = \text{pounds of } SO_2 \text{ to add.}$$

For example, suppose a desired $1\frac{1}{4}\%$ (1.25%) SO_2 brine is found by titration to be 1.1% in a 1,500 gallon tank. How many pounds of SO_2 should be added?

Solution:

$$SO_2 \text{ to add} = \frac{(1.25 - 1.10)(1,500 \text{ gal})(8.4 \text{ lbs/gal})}{100} = 18.9 \text{ lbs}$$

3. The mathematical expression for determining the amount of a certain grade of pectin equivalent to a given amount of another grade is:

$$\text{Amount of pectin to be used} = \frac{(\text{Wt of pectin being used})(\text{Grade of pectin being used})}{\text{Grade of pectin to be used}}$$

FORMULA
Level 1

Food Processing

A. Problems with Solutions (continued)

3. (continued)

We wish to know how much 60 grade pectin is equivalent to 12 pounds of 100 grade pectin. (Pectin is used in making jelly.)

Solution:

$$\text{Amount of 60 grade pectin} = \frac{(12 \text{ lbs})(100 \text{ grade})}{60 \text{ grade}} = 20 \text{ lbs}$$

B. Problems without Solutions

4. A bean canning line consists of eight different operations. The belt speeds are set for 106 No. 10 cans per minute. The line averages 7.67 hours per 8 hour shift. The individual belt production standards have been determined:

$$L = \left[\frac{T}{(S)(H)} \right] (1,000)$$

Determine the line standard for 1,000 cans of beans if there are the following in the crew: receiving 2, sizing 1, cutting 6, washing 4, filling 3, seaming 2, cooking 1 and one person warehousing.

DEFINITIONS:

- L: line standard expressed in standard hours per 1000 cans.
T: crew total standard hours per shift.
S: conveyor speeds in units per hour.
H: average hours run.

B. Problems without Solutions (continued)

5. A cold storage plant has a refrigeration capacity of 38 tons per day. How much time will be required to cool 42 tons of pears from 65° F to 35° F? Assume that heat transfer is not a problem and that the "heat of ripening" of pears to be 30,000 BTU per day per ton.

DEFINITIONS: 288,000 BTU must be removed to produce 1 TON of ice. Pears give off heat when they go through the chemical reactions of ripening.

6. This line, which has a rating of 0.0852 standard hours per 100 units produced, is filling 12 oz strawberry containers. The line needs to produce 60 units per minute. Using the formula $N = P \times S$ determine the number of people to be stationed at this belt.

DEFINITIONS:

N: number of crew required.
P: production required by crew in units.
S: 0.0852 standard hours per 100 units.

7. Determine the storage capacity in a 100,000 sq ft warehouse. This is a palletized operation with 4 ft by 4 ft pallets stacked four tiers high. There are 32 cases on a pallet. The total space per stack that must be allowed is 4 ft 2 in by 4 ft 2 in or 17.4 sq ft. The factor of inventory flexibility is 1.48 and the factor for unusual space is 1.54.

Space per case = $\frac{(\text{sq ft per stack})(\text{inventory flexibility factor})(\text{space factor})}{\text{cases of product per stack}}$

Capacity in cases = $\frac{\text{warehouse area}}{\text{space per case}}$

B. Problems without Solutions (continued)

8. If the test shows that SO_2 is too high, it will be necessary to add more lime. To determine the approximate amount of lime to add, use the following formula: (percent SO_2 found - percent SO_2 desired) multiplied by total gallons of brine and by 4.9 all divided by 100. Suppose the brine tests 1.45% SO_2 instead of the desired 1.25% in a 1,500 gallon tank. How many pounds of hydrated lime should be added?

9. You are assigned to make a fresh brine (12,000 gal) containing 15,000 ppm of sulfur dioxide using KHSO_3 which yields 0.67 sulfur dioxide by weight. How many pounds of KHSO_3 would you order?

$$\text{Pounds of } \text{KHSO}_3 = \frac{(\% \text{ desired } \text{SO}_2) (\text{gal brine}) (8.4 \text{ lbs/gal})}{(0.67) (\text{amount of free } \text{SO}_2)}$$

DEFINITIONS: ppm: parts per million. 8.4 lbs is the weight of the fresh brine per gallon.

10. If the normality of iodine used was 0.156 Normal and the brine sample size was 10 milliliters, what was the SDE?

$$\text{Formula: SDE} = \frac{(\text{normality of iodine solution}) (32,000)}{\text{ml of sample solution}}$$

DEFINITION: Sulfur dioxide equivalent (SDE) is defined as parts per million sulfur dioxide per ml of iodine solution used.

FORMULA
Level 2

Food Processing

B. Problems without Solutions (continued)

11. Determine the percent of protein in fresh sausage. If 32 ml of HCl acid at 0.1 N were used in testing a 2.0 gram sample of sausage, what is the percent of protein?

Formula:

$$\frac{(\text{ml acid}) (\text{N acid}) (0.014 \text{ mcg wt nitrogen}) (\text{factor}) (100)}{\text{g sample}} = \text{percent protein}$$

DEFINITION: Factor for converting nitrogen to protein is 6.25 for meats, fruits, nuts, wheat, wines and yeast.

12. Find the number of pounds per gallon from grams. Given 28.35 grams per ounce.

$$\text{Formula: } \frac{(3.785) (\text{g/ml}) (1000)}{(28.35) (16)} = \text{pounds per US gallon}$$

DEFINITIONS:

3.785 liters per US gallon

28.35 grams per ounce

16 ounces per pound

Water weighs 10 g per 10 ml at 3.98° C

13. After determining the amount of Ag NO₃ (to the nearest one-tenth ml) necessary to exactly reach the end point, calculate the salt content according to the following formula:

$$S = \frac{(T) (N) (0.05845) (100)}{V}$$

10 ml of pickle juice requires 41.7 ml of exactly 0.1 Normal Ag NO₃ to reach the end point. Calculate the salt content in the above sample.

DEFINITIONS:

0.05845: the factor necessary to convert the titration to NaCl and represents the number of grams of NaCl which will completely react with one ml of Normal Ag NO₃.

B. Problems without Solutions (continued)

13. DEFINITIONS: (continued)

100: the factor necessary to convert the results into percent by weight.

S: salt content

T: ml of Ag NO_3 required to reach the end point.

N: Normality of the Ag NO_3

V: sample size - either in ml or grams depending upon the terms in which the results are expressed.

14. To a 10.00 gram sample of green bean liquor add 40 ml 0.1 Normal Ag NO_3 . The filtrate requires 10.2 ml 0.1 Normal $\text{NH}_4 \text{CNS}$ to reach the end point. Using the formula from problem 13, determine the salt content in the given sample.

15. Small amounts of lime are added to wash water to raise the pH of the final pack. When 0.1 grams of lime is required to raise the pH of 1 liter of wash water to the proper final product level, how many pounds of lime are needed to treat 10,000 gallons of wash water?

$$P = \frac{(3.785) (G) (W)}{453.6}$$

DEFINITIONS:

P: pounds of lime

G: grams of lime needed for 1 liter

W: gallons of wash water to be treated

B. Problems without Solutions (continued)

16. Determine the parts per million of sulfur dioxide in a dehydrated potato sample. Titrations determined that 25 ml of NaOH at 0.01 N neutralized the SO₂ in 32.0 g of dehydrated potatoes. Use the following formula to determine the parts per million of SO₂.

$$\text{ppm SO}_2 = \frac{(\text{ml of base})(\text{N of base})(\text{meg wt SO}_2)(10^6)}{\text{g of sample}}$$

DEFINITIONS:

ppm: parts per million

meg wt or milliequivalent weight of SO₂ = 0.032

17. DEFINITION: Calculations of Drained Weights

Drained weights in the United States Standards are generally given for common container sizes. In the absence of specific recommended drained weights for other sizes of containers, the drained weight to be recommended may be figured on a proportionate basis, using No. 2 or No. 2 1/2 can sizes as a basis.

The equation would be:

$$\frac{\text{Recommended drained weight of product in 2 or 2 1/2 can}}{\text{Capacity of the No. 2 or No. 2 1/2}} = \frac{\text{Unknown recommended drained weight of can size that is not given}}{\text{Capacity of that container}}$$

Capacity of the No. 2 1/2 can is 29.75 ounces.

Capacity of the No. 2 1/2 glass jars is 28.375 ounces.

PROBLEM: Canned yellow clingstone peaches, sliced, heavy syrup. No. 2 1/2 glass jar. 18 ounces is recommended drained weight for 2 1/2 can "x" is unknown recommended drained weight for No. 2 1/2 glass jars.

B. Problems without Solutions (continued)

18. DEFINITION: Knowing the soluble solids of both the in-going fruit and the finished product, calculate the proportion of fruit to sugar as follows:

$$R = \frac{100 - P_s}{P_s - F_s} \text{ in which:}$$

R = Ratio of fruit to sugar

P_s = soluble solids of finished product (fruit plus sugar)

F_s = soluble solids of in-going fruit

PROBLEM: If the soluble solids of fresh peaches is 12.2, the soluble solids of the finished product (peaches plus dry sugar) is 31.8. Calculate the fruit-sugar ratio.

19. Determine if there is sufficient sulfurous acid to insure preservation. After measuring the OH, determine the parts per million (ppm) free sulfur dioxide by direct iodine titration.

Example: pH = 3.7

SO₂(ppm) = 5,000 (by iodine titration)

% SO₂ = (A) (refer to fig 4)

SO₂(ppm) = (B) x 5,000

= (C) ppm

B. Problems without Solutions (continued)

20. A new shipment of vinegar is to be tested for percent of acid. It takes 25.20 ml of NaOH (the base) at a Normality of 0.1000 (N) to neutralize 3.000 grams of vinegar. Using the formula below, what is the percent of acetic acid?

$$\frac{(\text{ml base}) (\text{N base}) (\text{meq wt acid}) (100)}{(\text{g sample})} = \text{percent acid}$$

DEFINITIONS:

meq: milliequivalent

meq wt acetic acid = 0.06005

21. Tin cans are rarely over 2% tin and the rest is steel. When a high content of tin is required, it is sometimes necessary to determine the amount of tin used in making the container. If 20 ml of $\text{Na}_2\text{S}_2\text{O}_3$ was used in the blank and 7.80 ml $\text{Na}_2\text{S}_2\text{O}_3$ was used in back titration, a 0.1 N $\text{Na}_2\text{S}_2\text{O}_3$ apply the following formula:

$$\frac{(\text{ml } \text{Na}_2\text{S}_2\text{O}_3 \text{ of blank} - \text{ml } \text{Na}_2\text{S}_2\text{O}_3 \text{ for back titration}) (\text{N } \text{Na}_2\text{S}_2\text{O}_3) (0.05935 \text{ meq wt stannous Sn}) (17.28)}{1} = \text{pounds Sn per base box.}$$

Both sides of a disc having a total of 8 square inches of tin plated surface were tested.

DEFINITION: Factor is based on use of sample area of 8 square inches compared to area of 67,720 square inches in a base box. Analysis by Bendix Method, Ind. Eng. Chem. Anal. Ed., 15, 501(1943).

B. Problems without Solutions (continued)

22. You are required to determine an economic lot size for end bearings with an annual requirements of 480 units, restocking costs of \$25, annual carrying cost ratio of 21% and a unit cannery cost of \$15.

Economic lot size formula is: $Q = \sqrt{\frac{2 AR}{CU}}$

DEFINITIONS:

- Q: economic lot size order in units
- A: annual requirements in units
- R: restocking cost in dollars
- C: annual carrying cost ratio
- U: unit factory cost in dollars

FORMULA
Level 1

Food Processing

Complete Solutions to B Problems

4.	No. in Crew	Total Hours per 8 Hour Day
Receiving	2	16
Sizing	1	8
Cutting	6	48
Washing	4	32
Filling	3	24
Seaming	2	16
Cooking	1	8
Warehousing	1	8
		<u>160 hrs</u>

$$L = \frac{(160 \text{ hrs})(1,000)}{(106 \text{ cans/min})(60 \text{ min/hr})(7.67 \text{ hrs})}$$

$$= \frac{160,000 \text{ hrs}}{48781.2 \text{ cans}} = 3.28 \text{ standard hours per 1,000 cans produced}$$

$$5. \text{ Cooling time} = \frac{(65-35)(42 \text{ tons})(30,000 \text{ BTU/day/ton})(1 \text{ day})}{(38 \text{ tons/day})(288,000 \text{ BTU/ton})}$$

$$= \frac{(30)(42)(30,000)}{(38/\text{day})(288,000)} = \frac{37,800,000}{10,994,000/\text{day}} = 3.45 \text{ days}$$

$$6. \quad S = 0.0852 \text{ standard hrs per 100 units}$$

$$S = 0.000852 \text{ standard hrs per unit}$$

$$N = (60 \text{ units/min})(60 \text{ min/hr})(0.000852 \text{ hrs/unit})$$

$$N = 3.07 \quad \text{Three people should be stationed on this short belt.}$$

$$7. \text{ Space per case} = \frac{(17.4 \text{ sq ft/stack})(1.48)(1.54)}{(32 \text{ cases/pallet})(4 \text{ pallets/stack})}$$

$$= \frac{39.658 \text{ sq ft}}{128 \text{ cases}} = 0.31 \text{ sq ft/case}$$

$$\text{Capacity} = \frac{100,000 \text{ sq ft}}{0.31 \text{ sq ft/case}} = 323,000.00 \text{ cases}$$

$$8. \text{ Lime to add} = \frac{(1.45 - 1.25)(1,500 \text{ gals})(4.9 \text{ lbs/gal})}{100} = 14.7 \text{ lbs}$$

FORMULA
Level 1

Food Processing

Complete Solutions to B Problems (continued)

$$9. \text{ KHSO}_3 = \frac{(0.015)(12,000 \text{ gal})(8.4 \text{ lbs/gal})}{0.67} = 2,256.7 \text{ lbs}$$

$$10. \text{ SDE} = \frac{(0.156)(32,000)}{10} = 499.2 \text{ or } 500 \text{ SDE on a practical level}$$

Level 2

$$11. \text{ Protein} = \frac{(32 \text{ ml})(0.1)(0.014)(6.25)(100)}{2.0 \text{ g}} = 14\%$$

$$12. \text{ Pounds per US gallon} = \frac{(3.785 \text{ l/gal})(10\text{g}/10 \text{ ml})(1000 \text{ ml/l})}{(28.35 \text{ g/oz})(16 \text{ oz/lb})}$$
$$= 8.34 \text{ lb/gal}$$

$$13. \text{ S} = \frac{(41.7 \text{ ml})(0.1)(0.05845 \text{ g/ml})(100)}{10 \text{ ml}} = 2.44 \text{ grams per 1 ml}$$

$$14. \text{ 0.1 Normal AgNO}_3 \text{ in the reaction} = 40 \text{ ml} - 10.2 \text{ ml} = 29.8 \text{ ml}$$
$$\text{S} = \frac{(29.8 \text{ ml})(0.1)(0.05845 \text{ g/ml})(100)}{10 \text{ g}} = 1.74\%$$

$$15. \text{ P} = \frac{(3.785)(0.1)(10,000)}{453.6} = 8.34 \text{ lbs of lime}$$

$$16. \text{ Ppm SO}_2 = \frac{(25)(0.01)(0.032)(10^6)}{32.0} = 250$$

$$17. \text{ D} = \text{recommended (drained weight)}$$

$$\frac{18 \text{ oz}}{29.75 \text{ oz}} = \frac{\text{D}}{28.375 \text{ oz}}$$

$$(29.75)\text{D} = 510.75 \text{ oz}$$

$$\text{D} = 17.2 \text{ oz}$$

FORMULA
Level 2

Food Processing

Complete Solutions to B Problems (continued)

$$18. \quad R = \frac{100 - 31.8}{31.8 - 12.2} = \frac{68.2}{19.6} = 3.48$$

If the product was labeled 3.5 to 1 ratio, the actual proportion of fruit to sugar (3.48 to 1) is sufficiently close to consider as meeting requirements.

$$19. \quad A = 1.2\%$$

$$B = 0.012$$

$$C = 60 \text{ ppm}$$

$$20. \quad \text{Acid} = \frac{(25.20)(0.1)(0.06005)(100)}{3.0} = 5.0\%$$

$$21. \quad T = \text{pounds of tin per base box}$$

$$T = (20 - 7.8)(0.1)(0.05935)(17.28) = 1.25$$

$$22. \quad Q = \sqrt{\frac{(2)(480 \text{ units})(\$25.00)}{(0.21/\text{unit})(\$15.00)}}$$

$$Q = \sqrt{\frac{24,000 \text{ units}^2}{3.15}}$$

$$Q = \sqrt{7619} \text{ units}$$

$$Q = 87 \text{ units}$$

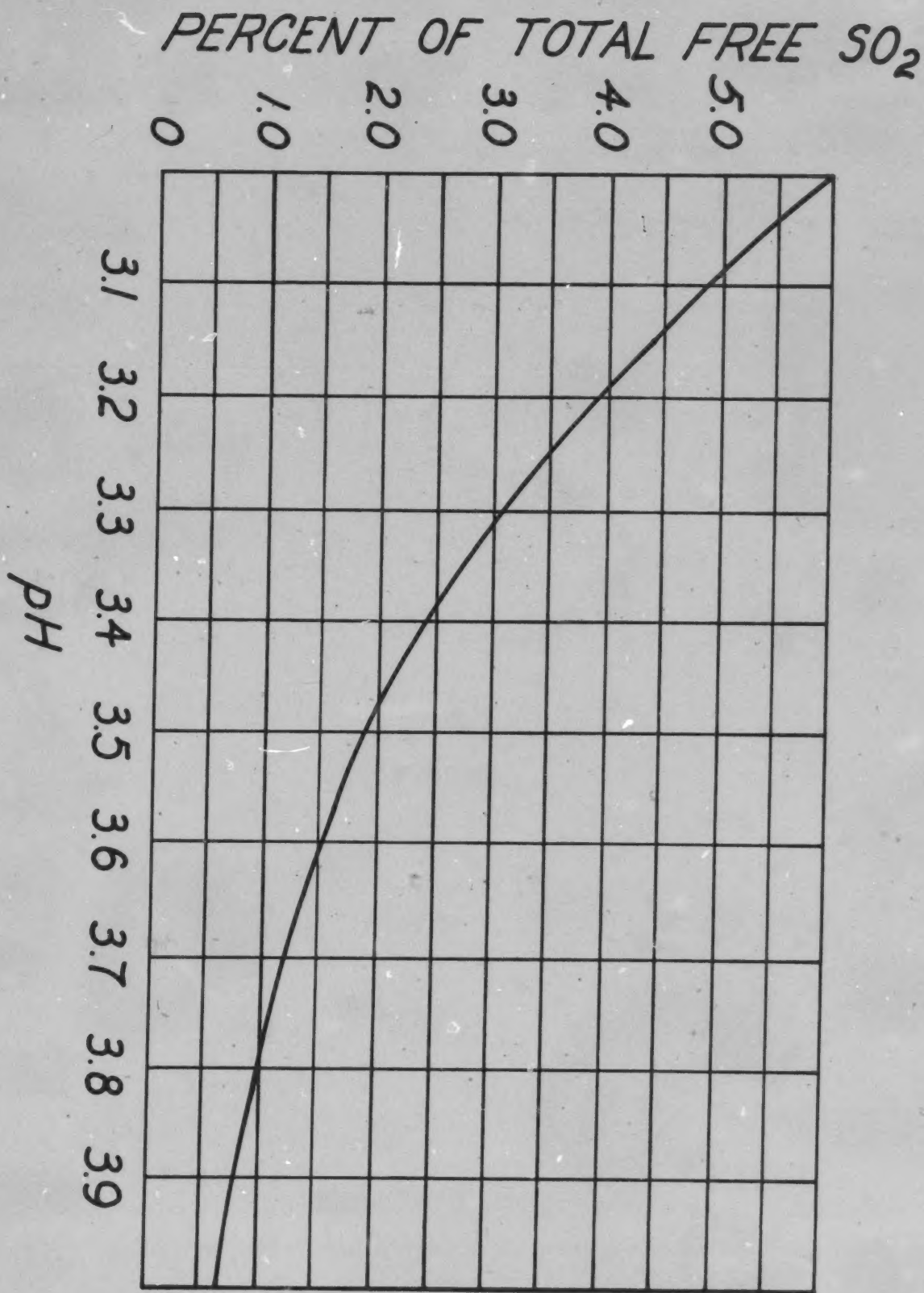
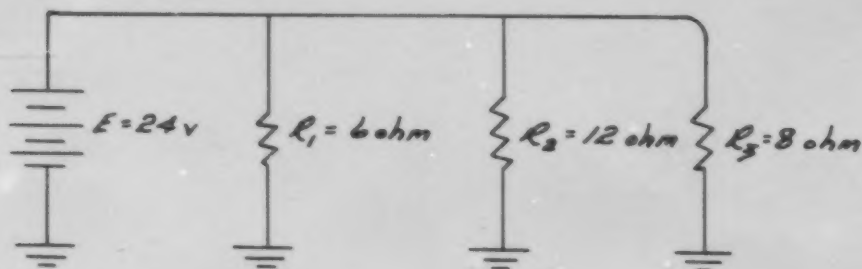


Figure 4. Effect of pH on distribution of sulfurous acid in solution.

A. Problems with Solutions

1. In the electrical circuit indicated by the figure, what is the total resistance?



Solution:

In a parallel circuit, the total resistance at a group of resistances is equal to the reciprocal of the sum of the reciprocals of the resistances.

$$\begin{aligned} R_t &= \frac{1}{1/R_1 + 1/R_2 + 1/R_3} \\ &= \frac{1}{1/6 + 1/12 + 1/8} \\ &= \frac{1}{\frac{4 + 2 + 3}{24}} = \frac{24}{7} = \frac{8}{3} \text{ ohms} \end{aligned}$$

2. If a thermometer reads 30° C, what would a Fahrenheit thermometer indicate in the same location?

Solution:

$$\begin{aligned} ^\circ F &= 9/5 ^\circ C + 32^\circ = (9/5)(30)^\circ + 32^\circ \\ &= 54^\circ + 32^\circ = 86^\circ \end{aligned}$$

B. Problems without Solutions

3. What is the amount of lift which can be obtained from an airplane wing having an area of 180 sq ft, at a velocity of 120 mph, at an altitude of 1000 ft, with a C (lift coefficient) of 0.4 at a 4° angle of attack?

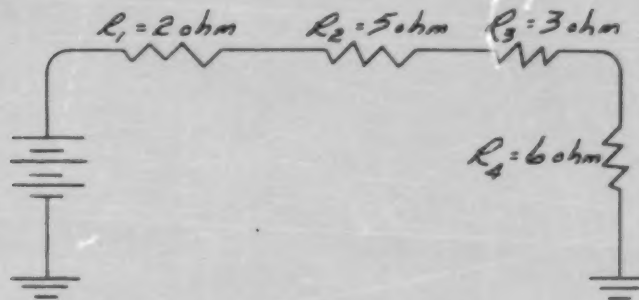
The formula for lift is $L = C_L \frac{\rho}{2} V^2 S$ where C_L = lift coefficient; ρ = mass density, slugs/cu ft. In this case $\rho = 0.002309$. V = velocity of wind relative to the body, ft per second; S = Airfoil area sq ft. To convert from slugs to lbs, multiply by 32.1740

4. Find the total resistance in a circuit having resistors of 4, 8, and 12 ohms connected in parallel. $R_t = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$ for resistors in parallel.

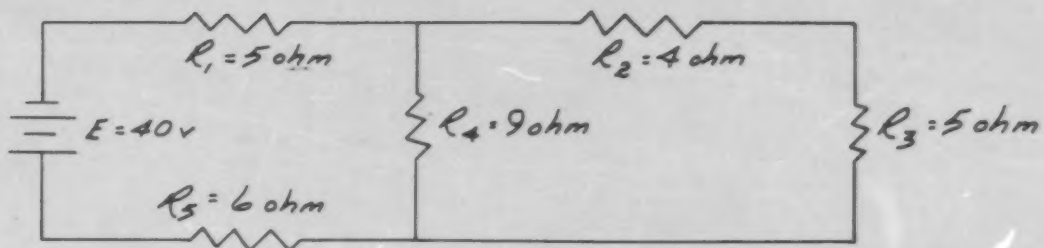
5. What is the resistance in a circuit when resistors of 3, 6, and 8 ohms are connected in a series? $R_t = R_1 + R_2 + R_3 \dots$, for resistors in series.

B. Problems without Solutions (continued)

6. In the electrical circuit indicated by the figure, what is the total resistance?



7. In the circuit indicated by the figure, what is the total resistance?



8. How much electrical power is consumed by a motor which draws a current of 10 amp in a 12 volt system? $W = I E$ where W = Power (watts), I = Current (amps) and E = Emf(volts).

B. Problems without Solutions (continued)

9. Compute the thrust of a jet engine mounted in an airplane flying at 900 feet per second. The engine takes in 200 lb per second of air and burns 4 lb per second of fuel. The velocity of the jet exhaust is 2,100 ft per second with respect to the engine.

$$F = \frac{M}{g} (V_2 - V_1)$$

M = Mass rate of flow (lbs/sec)
 g = Acceleration of gravity
 V_2 = Final velocity
 V_1 = Initial velocity

The following formulas are useful in considering the next three problems.

$$X_C = \frac{1}{2 \pi f C} \quad X_L = 2 \pi f L \quad f = \frac{1}{2 \pi \sqrt{LC}}$$

where X_C = capacitive reactance, ohms
 X_L = inductive reactance, ohms
 f = frequency, cps (cycles per second)
 C = capacitance, farads
 L = inductance, henrys
 π = 3.14

10. What is the capacitive reactance in a circuit when the frequency is 400 cps and the capacitance in the circuit is 0.5 μ f.

B. Problems without Solutions (continued)

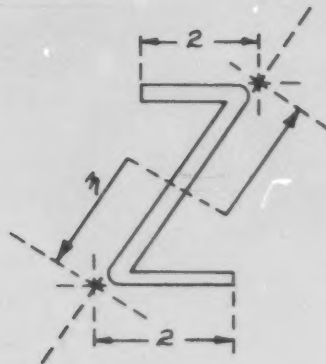
11. If a circuit contains 10 mh (millihenry) of inductance and the frequency is 400 cps, what is the inductive reactance?

12. Find the frequency where a circuit is resonant when the capacitance is $0.5 \mu f$ and the inductance is 10.0 mhenrys.

Information for Problems of Layout in Bending Material

Bending is a forming operation used extensively in aircraft work. A curved corner requires less material than a square corner, and it must be possible to cut the stock correctly before the bending is to be done.

The BEND ALLOWANCE (BA) is that part required to make a bend over a given inside radius. It is the distance between bend tangent lines before the bend is made.



B. Problems without Solutions (continued)

Information for Problems of Layout in Bending Material (con't)

The amount of material saved by bending around a radius is called SETBACK. This needs to be known since the length (developed width) of material we shall see is equal to the sum of the mold line distances minus the setback.

To find the setback, both the BA and the "x" distance must be known. The x distance may be found by using a k chart (this is a table of values for the tangent function; k is the tangent of one half of the bend angle). The formula for finding x distance = $(K)(R + T)$ where R is the inside radius bend, and T is the thickness of the metal.

An empirical formula has been derived suitable for determining BA.

$$BA = (0.01743R + 0.0078T) (\text{number of degrees in bend})$$

When x distance and BA are known, then, $\text{Setback} = 2x - BA$

The amount of material required to make the part is called the DEVELOPED WIDTH (DW). So, $DW = MLD_t - SB_t$, where MLD_t = total mold line distance and SB_t = sum of the setbacks.

Level 3

13. Compute the bend allowance for a 60° bend with a $1/4$ in radius when the thickness is 0.050 inch.

B. Problems without Solutions (continued)

14. Compute the developed width of a piece of aluminum--alloy sheet which is to be formed into a Z section with two bends of 110° each around a radius of $3/8$ in when the thickness of the metal is 0.060 in, the mold-line distance for each flange is 2 in, and the mold-line distance for the inside flat section is 3 in.

Note: Tangent of $(0.5)(110^\circ) = \text{tangent } 55^\circ = 1.428$



Complete Solutions to B Problems

3. To solve the problem, the velocity is converted into feet per second; 120 mph = 176 ft/sec. Next, by reference to NASA standard atmospheric tables, it is found that the value of ρ (density) is 0.002309.

Substituting values,

$$L = (0.4) \frac{[(0.002309) (32.1740)] \text{ lb} (176 \text{ ft})^2 (180 \text{ ft})}{(\text{cu-ft}) (2) (\text{sec})^2}$$

$$= 82,843.1 \frac{\text{lbs-ft}}{\text{sec}}$$

4. $R_t = \frac{1}{1/R_1 + 1/R_2 + 1/R_3} = \frac{1}{1/4 + 1/8 + 1/12}$

$$= \frac{1}{\frac{6 + 3 + 2}{24}} = \frac{24}{11} \text{ ohms}$$

5. Since these are in series,

$$R_t = 3 \text{ ohms} + 6 \text{ ohms} + 8 \text{ ohms} = 17 \text{ ohms}$$

6. When the resistances (loads) are connected in series, the total resistance is the sum of the resistances.

$$R_t = 2 \text{ ohms} + 5 \text{ ohms} + 3 \text{ ohms} + 6 \text{ ohms} = 16 \text{ ohms}$$

7. This is a combination of series and parallel connections.

$$6 + \frac{1}{\frac{1}{9} + \frac{1}{5 + 4}} + 5 = 6 + \frac{1}{2/9} + 5$$

$$= 11 + 9/2 = 15 \frac{1}{2} \text{ ohms}$$

8. $W = I E$

$$\text{So, } W = (10 \text{ amps}) (12 \text{ volts}) = 120 \text{ watts}$$

FORMULAS
Level 2

Aviations Mechanics

Complete Solutions to B Problems (continued)

9. From the equation $F = \frac{M}{g} (V_2 - V_1)$,

$$F = \frac{204 \text{ lb} (2100 \text{ ft} - 900 \text{ ft}) \text{ sec}^2}{32.2 \text{ ft sec sec}} = 7600 \text{ lb}$$

10. $X_C = \frac{1}{2 \pi f C} = \frac{1}{(2) (3.14) (400 \text{ cps}) (0.0000005 \text{ f})} = 796.18 \text{ ohms}$

11. $X_L = 2 \pi f L$

$$X_L = (6.28) (400 \text{ cps}) (0.010 \text{ h})$$

$$= 25.12 \text{ ohms}$$

12. $f = \frac{1}{2 \pi \sqrt{LC}} = \frac{1}{(6.28) (\sqrt{(0.010 \text{ h}) (0.0000005 \text{ f})})}$
2,251.9 cps

Level 3

13. $BA = [(0.01743R + 0.0078T)] (60^\circ)$
 $= [(0.01743) (0.25) + (0.0078) (0.050)] (60^\circ)$
 $= [(0.0043575 + (0.000390))] (60^\circ)$
 $= (0.0047475) (60^\circ)$
 $= 0.28485 \approx 0.285 \text{ in}$

14. To utilize $DW = M L D_t - S B_t$, we must find $SB = 2X - BA$

where $X = (K) (R + T)$, (see previous material) and $BA = (0.01743R + 0.0078T) (110)$

$$BA = [(0.01743) (0.375) + (0.0078) (0.060)] (110)^\circ$$

$$= [(0.00653625 + 0.000608)] (110)^\circ$$

$$= (0.00714425) (110)^\circ$$

$$= 0.78586750 \text{ in}$$

$$X = (1.428) (0.375 \text{ in} + 0.060 \text{ in})$$

$$= 0.621180 \text{ in}$$

FORMULAS
Level 3

Aviation Mechanics

Complete Solutions to B Problems (continued)

14. (con't)

$$SB = (2)(0.621180 \text{ in}) - 0.78586750 \text{ in}$$

$$= 1.24236 \text{ in} - 0.78586750 \text{ in}$$

$$\approx 0.45649 \text{ in}$$

$$DW = (2 \text{ in} + 3 \text{ in} + 2 \text{ in}) - (2)(0.45649 \text{ in}) \approx 6.087 \text{ in}$$

FORMULAS
Level 1

Forest Products

A. Problems with Solutions

1. Probably the most realistic formula for estimating the number of board feet of lumber in a log is called the International Log Rule. This rule assumes a constant taper of 0.5 in per 4 ft of length. It corrects for slabs as a function of log diameter and it assumes 1/4 in and 1/16 in for saw kerf and shrinkage, respectively. Incidentally this rule also assumes that all the lumber is sawed into 1 inch boards. Using the International Rule, calculate the number of board feet in 16 foot logs whose d.i.b. are 10, 20 and 40 inches.

$$\text{International Rule: Board feet} = (0.22)(D^2) - (0.71)D$$

Solution:

For 10 inch diameter log	
Small end diameter in inches	Board feet in each 4 foot section
10	14.9
10.5	16.8
11	18.8
11.5	20.9
Total board feet in 16 ft, 10 in diameter log	
	71.4

For 20 inch diameter log	
Small end diameter in inches	Board feet in each 4 foot section
20	73.8
20.5	77.9
21	82.1
21.5	86.4
Total board feet in 16 ft, 20 in diameter log	
	320.2

For 40 inch diameter log	
Small end diameter in inches	Board feet in each 4 foot section
40	323.6
40.5	332.1
41	340.7
41.5	349.4
Total board feet in 16 ft, 40 in diameter log	
	1,345.8

B. Problems without Solutions

2. There is no "good" method for converting from board-feet log scale to board-feet of lumber. The term "board foot" is rather ridiculous when applied to log volume; however, it is applied with persistence. A rule of thumb formula (which provides a close approximation to Scribner's log rule) for the board feet of lumber in a 16 foot log is:

$$(0.8)(D - 1)^2 - \frac{D}{2}$$

Use this formula to estimate the number of board feet in 16 foot logs whose d.i.b. are 10, 20 and 40 inches.

B. Problems without Solutions (continued)

3. Another rule for estimating the number of board feet of lumber (which assumes a 4 inch slab and a 5/16 inch saw kerf) is Doyle's Rule. This rule results in a large underscale for small logs and overscale for large logs. Use this formula to estimate the board feet of lumber in 16 foot logs whose d.i.b. are 10, 20 and 40 inches. Compare with the Rule of Thumb formula used in the previous problem. D is diameter inside bark in inches.

Doyle's Rule:
Board feet = $\left(\frac{D - 4}{4}\right)^2$ (Length, ft)

Rule of Thumb:
Board feet = $(0.8)(D - 1)^2 - \frac{D}{2}$

B. Problems without Solutions (continued)

4. In measuring the cubic foot contents of logs it is convenient to assume a circular cross-section, determine this cross-sectional area (Basal area) in square feet, and then simply multiply the Basal area by the length in feet to obtain the total volume. Since logs are not perfect cylinders, various methods have been devised for approximating the volume. Three common formulae are:

$$\text{Huber's Volume} = (B_{\frac{1}{2}}) (L)$$

$$\text{Smalian's Volume} = \left(\frac{B + b}{2} \right) (L)$$

$$\text{Newton's Volume} = (B + 4B_{\frac{1}{2}} + b) \left(\frac{L}{6} \right)$$

Where $B_{\frac{1}{2}}$ = Basal area at log midpoint

B = Basal area at the large end

b = Basal area at the small end

L = Log length

Compute the volume of a 16 foot log whose diameter (d.i.b.) is 20 in on the small end, 21 in at the midpoint and 22 in on the large end -- using each of the above formulae.

FORMULAS

Forest Products

Level 1

Complete Solutions to B Problems

2. For 10 inch diameter log

$$\begin{aligned}\text{Number of board feet} &= (0.8)(10 - 1)^2 - \frac{10}{2} \\ &= 64.8 - 5 \\ &= 59.8\end{aligned}$$

For 20 inch diameter log

$$\begin{aligned}\text{Number of board feet} &= (0.8)(20 - 1)^2 - \frac{20}{2} \\ &= 288.8 - 10 \\ &= 278.8\end{aligned}$$

For 40 inch diameter log

$$\begin{aligned}\text{Number of board feet} &= (0.8)(40 - 1)^2 - \frac{40}{2} \\ &= 1,216.8 - 20 \\ &= 1,196.8\end{aligned}$$

3. For 10 inch diameter log

$$\text{Number of board feet} = \left(\frac{10 - 4}{4}\right)^2 (16) = 36$$

For 20 inch diameter log

$$\text{Number of board feet} = \left(\frac{20 - 4}{4}\right)^2 (16) = 256$$

For 40 inch diameter log

$$\text{Number of board feet} = \left(\frac{40 - 4}{4}\right)^2 (16) = 1,296$$

4. Basal area in sq ft = $\frac{(\pi)(d.i.b.)^2}{(4)(144 \text{ sq in/sq ft})}$

$$b = \frac{(3.14)(20 \text{ in})^2}{(4)(144 \text{ sq in/sq ft})} = 2.18 \text{ sq ft}$$

$$B_{1/2} = \frac{(3.14)(21 \text{ in})^2}{(4)(144 \text{ sq in/sq ft})} = 2.41 \text{ sq ft}$$

$$B = \frac{(3.14)(22 \text{ in})^2}{(4)(144 \text{ sq in/sq ft})} = 2.64 \text{ sq ft}$$

Using Huber's equation

$$\text{Volume} = (B_{1/2})(L) = (2.41 \text{ sq ft})(16 \text{ ft}) = 38.56 \text{ cu ft}$$

Using Smalian's equation

$$\begin{aligned}\text{Volume} &= \left(\frac{B + b}{2}\right)(L) = \left(\frac{2.64 \text{ sq ft} + 2.18 \text{ sq ft}}{2}\right)(16 \text{ ft}) \\ &= 38.56 \text{ cu ft}\end{aligned}$$

Using Newton's equation

$$\begin{aligned}\text{Volume} &= (B + 4B_{1/2} + b)\left(\frac{L}{6}\right) \\ &= [2.64 \text{ sq ft} + (4)(2.41 \text{ sq ft}) + 2.18 \text{ sq ft}]\left(\frac{16 \text{ ft}}{6}\right) \\ &= 38.56 \text{ cu ft}\end{aligned}$$

A. Problems with Solutions

1. If the price of an object is increased or decreased, its new price P may be expressed as $P = P_o (1 + r)$, where P_o is the original price and r is expressed as a percentage. The quantity r is positive if the price is increased and negative if the price is decreased. Find the price of an object costing originally \$20 after the price has been increased by 30 percent and after it has been decreased by 30 percent.

Solution:

(a) $P = P_o (1+r)$	(b) $P = P_o (1-r)$
$P = (\$20) (1 + 0.30)$	$P = (\$20) (1 - 0.30)$
$P = \$26$	$P = \$14$

2. Write the formula in dollars for the charge, C , for a telephone call between two towns if the conversation lasted n minutes, n being greater than 3. Assume that the charge for the first 3 minutes is 55¢, and each additional minute costs 10¢.

Solution:

$C = \$0.55 + (n-3) (\$0.10)$	$n > 3$
$C = \$0.25 + (\$0.10)n$	$n > 3$

3. Use the formula $E = SR + b$ and determine R .

A clothing salesman received a total weekly salary of \$130. If his base salary was \$80 and he sold \$1,000 worth of clothing, how much was his rate of commission? (Express the solution as a per cent.)

Solution:

$$\begin{aligned} E &= SR + b \\ \$130 &= (\$1,000)R + \$80 \\ \$50 &= (\$1,000)R \\ 5\% &= R \end{aligned}$$

A. Problems with Solutions (continued)

The true rate of interest on installment purchases may be made quickly using:

$$R = \frac{2 M I}{P (N + 1)}$$

R = true rate of interest

M = number of periods per year

I = interest (difference between cash price and credit price)

P = principal of loan

N = number of payments

4. Benny Beasley borrowed \$500 from his bank. If the bank charges 8% interest on personal loans and requires repayment of the loan in 10 equal monthly installments of \$53.33, what is the true rate of interest on the loan?

Solution:

$$\text{Total Payment} = (10 \text{ payments}) (\$53.33/\text{payment}) = \$533.30$$

$$\begin{aligned} \text{Interest} &= \text{Total Payment} - \text{Principal} \\ &= \$533.30 - \$500.00 = \$33.30 \end{aligned}$$

$$R = \frac{2 M I}{P (N + 1)} = \frac{(2) (12) (\$33.30)}{(\$500) (10+1)} = \frac{\$799.2}{\$5500} = 0.1453 = 14.53\%$$

B. Problems without Solutions

5. Use the formula $C = PN + d$ and determine d .

A cement company has a fixed charge on deliveries. If the total cost is \$90 and a mason orders 1,000 bricks at \$0.08 each, how much is the delivery charge?

Solution:

6. Lucky John's Furniture Store charges "low bank rates" of 8% of the principal on all new furniture sales. What is the true rate of interest on a new refrigerator that sells for a cash price of \$425 if the principal and interest are paid off in 10 monthly installments? (Use formula from problem 4)

Solution:

7. Using the formula $C = PN + d$, determine P .

A lumber company charges \$6 for a delivery. A carpenter orders 50 sheets of 4 foot by 8 foot paneling. The total cost was \$456. How much was the price per panel?

Solution:

B. Problems without Solutions (continued)

8. Nolan's Boat Shop sells a special model plastic boat at a cash price of \$459. If the same boat is purchased for \$50 down and 12 monthly payments of \$38, what rate of interest is the customer paying?

Solution:

9. Write the formula in dollars for the charge, C , for borrowing a book from a lending library for n days, n being greater than 7. Assume that the charge for the first week is 35¢ and each additional day costs 3¢.

Solution:

10. The total cost Q of producing x units of a commodity is given by the formula:

$$Q = \frac{19}{6} + \frac{x}{8} + \frac{x^2}{18}$$

How many dollars does it cost to produce 50 units?

Solution:

TRADE FORMULAS
Level 1

Clerical

Complete Solutions to B Problems

5. $C = PN + d$
 $\$90 = (\$0.08/\text{brick})(1,000 \text{ bricks}) + d$
 $\$90 - \$80 = d$
 $\$10 = d$
6. First we must find the total interest
 $\text{Interest} = (\$425)(\$0.08) = \$34.00$
 Using the formula $R = \frac{2MI}{P(N+1)}$

$$R = \frac{(2)(12)(\$34)}{(\$425)(10+1)} = \frac{\$816}{\$4,675} = 0.1745 = 17.45\%$$
7. $C = PN + d$
 $\$456 = P(50 \text{ panels}) + \$6$

$$\frac{\$450}{50 \text{ panels}} = P$$

 $\$9 \text{ per panel} = P$
8. Total Principal on Loan = $\$459 - \$50 = \$409$
 $\text{Interest} = (\$38/\text{payment})(12 \text{ payments}) - \$409$
 $= \$456 - \$409 = \$47$
 Using the formula $R = \frac{2MI}{P(N+1)}$

$$R = \frac{(2)(12)(\$47)}{(\$409)(12+1)} = \frac{\$1,128}{\$5,317} = 0.2121 = 21.21\%$$
9. $C = \$0.35 + (\$0.03)(n-7) \quad n > 7$
 $C = \$0.14 + (\$0.03)n \quad n > 7$
10. $Q = \frac{19}{6} + \frac{x}{8} + \frac{x^2}{18}$
 $Q = \frac{19}{6} + \frac{50}{8} + \frac{2500}{18}$
 $Q = \frac{228 + 450 + 10,000}{72}$
 $Q = \frac{10,678}{72} = \$148.31$

USEFUL FORMULAE

TORQUE IN FT LBS

$$\text{FT LBS} = \frac{5252 \times \text{HP}}{\text{RPM}} \text{ OR } \frac{\text{BHP} \times 5252}{\text{RPM}}$$

HP ON DYNAMOMETER

$$\text{HP} = \frac{\text{LOAD (SCALE BEAM)} \times \text{RPM}}{\text{DYNAMOMETER CONSTANT}}$$

FUEL RATE IN LBS/HR

$$\text{FUEL RATE} = \frac{\text{LBS FUEL} \times 60}{\text{TIME IN MINUTES}}$$

- 1 HP = 0.746 KW = 33,000 FT LBS PER MINUTE
- 1 KW = 1.34 HP = 44,254 FT LBS PER MINUTE
- 1 IN OF MERCURY = 0.4912 PSI
- 1 IN OF WATER = 0.0361 PSI
- 1 U.S. GALLON = 231 CU IN
- 1 IMPERIAL GALLON = 1.2 U.S. GALLON

SEA LEVEL HP

60°F And 29.92 In/Hg
Barometric Pressure

500 FEET HP (SAE)

85°F And 29.38 In/Hg
Barometric Pressure

NATURALLY ASPIRATED AND SUPERCHARGED ENGINE DERATION

- 3% Each 1000 Ft ABOVE SEA LEVEL
- 1% Each 10°F RISE TEMPERATURE

TURBOCHARGED ENGINE DERATION

- 4% Each 1000 Ft ABOVE ALTITUDE RATING

A. Problems with Solutions

POWER AND PERFORMANCE TERMINOLOGY

POWER is the rate at which work is done.

HORSEPOWER is a unit of power equal to 33,000 ft lb per minute.

INDICATED HORSEPOWER is the total horsepower actually developed on the pistons in the engine.

$$\text{Indicated Horsepower} = \text{Friction Horsepower} + \text{Brake Horsepower}$$

FRICTION HORSEPOWER is the horsepower used in overcoming friction in the mechanical parts of the engine and in those processes associated with induction of the fuel and air and disposition of exhaust gases. Friction horsepower varies with RPM, oil temperature, throttle setting, etc.

BRAKE HORSEPOWER is the horsepower delivered at the engine flywheel and is equal to Indicated Horsepower less Friction Horsepower.

$$\begin{array}{l} \text{Brake Horsepower} = \frac{\text{BMEP} \times \text{RPM} \times \text{Displacement}}{792,000} \\ (4 \text{ cycle}) \end{array}$$

$$\begin{array}{l} \text{Brake Horsepower} = \frac{\text{BMEP} \times \text{RPM} \times \text{Displacement}}{396,000} \\ (2 \text{ cycle}) \end{array}$$

BRAKEMEAN EFFECTIVE PRESSURE (BMEP) may be defined as a theoretical constant pressure exerted during the power stroke to produce a power output equal to the Brake Horsepower. BMEP does not exist as a reality. It is strictly a calculated value of the equivalent constant pressure an engine would have to produce to develop a certain Brake Horsepower.

$$\begin{array}{l} \text{BMEP (in PSI)} = \frac{792,000 \times \text{BHP}}{\text{Disp. (cu in)} \times \text{RPM}} \\ (4 \text{ cycle}) \end{array}$$

$$\begin{array}{l} \text{BMEP (in PSI)} = \frac{396,000 \times \text{BHP}}{\text{Disp. (cu in)} \times \text{RPM}} \\ (2 \text{ cycle}) \end{array}$$

A. Problems with Solutions (continued)

1. Engine Torque: The output of a motor can be expressed in either torque or horsepower. In automotive calculations, it is customary to use the torque value since it simplifies the calculations. The relationship between torque and horsepower is given by the formula:

$$T = \frac{(5252) (HP)}{RPM} \quad \text{where } T = \text{torque in pounds-feet}$$

Hp = horsepower
RPM = engine speed in revolutions per min

What is the torque of an engine that develops 80 HP at 2500 RPM?

Solution:

$$T = \frac{(5252) (80 \text{ hp})}{2500 \text{ rpm}} \approx 168 \text{ lbs-feet (nearest unit)}$$

2. Axle Torque: The torque available at the driving axle is the engine torque multiplied by gear reduction through the transmission and axle.

$$T_a = (T) (R_t) (R_{ta}) (R_a)$$

where T_a = axle torque

R_t = gear reduction through transmission for the particular gears engaged

R_a = axle gear reduction

R_{ta} = gear reduction through auxiliary transmission

T = engine torque

What is the rear axle torque in low gear on a vehicle having 168 lb/feet engine torque with a transmission ratio of 6.8:1, an auxiliary ratio of 1.5:1, and an axle of 5.14:1?

Solution:

$$T_a = (168) (6.8) (1.5) (5.14) \approx 8,808 \text{ lb/ft (nearest unit)}$$

3. Find the required horsepower to overcome air resistance if the truck has a speed of 40 mph, and front dimensions of 8' wide and 12.5' high.

Solution:

$$\text{Required Hp} = \frac{(\text{MPH}^3) (\text{Area})}{\text{constant}}, \text{ constant is } 150,000$$

$$\frac{(40^3) (8) (12.5)}{150,000} = \frac{(64,000) (100)}{150,000} = 42.66 \text{ Hp}$$

B. Problems without Solutions

4. Engine speed: The engine speed is obtained by multiplying 168 by the ratio of each unit in the power train by the miles per hour and dividing this product by the rolling radius of the tire.

$$\text{RPM} = \frac{(168) (R) (\text{MPH})}{r}$$
 Find the engine speed where the overall gear reduction is 10, vehicle speed is 15 MPH, and rolling radius of driving tire is 19.5 inches.

5. Air Resistance: The air resistance against a vehicle is a force in pounds equal to 0.0025 times the miles per hour squared times the frontal area.

$$\text{AR} = [0.0025 (\text{MPH})^2] (\text{FA})$$
 What is the air resistance of a vehicle traveling 40 miles per hour and having a frontal area of 80 sq ft?

6. Horsepower required to overcome air resistance: The horsepower required to overcome air resistance is the speed in mph, cubed, times the frontal area in square feet divided by 150,000.

$$\text{HP} = \frac{(\text{MPH})^3 (\text{FA})}{150,000}$$
 What is the horsepower required to overcome air resistance of a vehicle traveling 40 mph and having a frontal area of 80 square feet?

B. Problems without Solutions (continued)

7. Clutch Torque Capacity: Clutch torque capacity is the sum of friction surfaces times coefficient of friction of linings, times the total effective pressure on linings times the radius of gyration of linings.

$$T = (N) (f) (P) (K)$$

where T = torque in lbs ft

N = number of friction surfaces

f = coefficient of friction of linings (0.25 average slip)

P = total effective pressure on linings

K = radius of gyration of lining in feet

What is the torque capacity of a clutch having a driven disc 15.5" O.D. and 9" I.D.? The disc has two working surfaces. Pressure on disc is 5,000 lbs and coefficient of friction 0.25 and K = 0.5281.

8. If the bhp of a 6-cylinder engine is 210 bhp at 1,800 rpm, the piston area for one piston is 18.66 sq in, and the length of the stroke is 6 in, find the bmep. Use the formula:

$$bmep = \frac{(bhp) (33,000)}{(P) (L) (N)}$$

where P = one piston head area

L = length of stroke in ft

N = the number of power strokes per minute of all pistons
or N = (rpm) (3) = 5400

FORMULAS
Level 2

Diesel Mechanic

B. Problems without Solutions (continued)

9. The axle ratio determines the road speed of the truck in direct drive. The ratio selected affects performance, fuel economy, and life. The formula for axle ratio is

$$\text{Axle Ratio} = \frac{(60) (\text{RPM})}{(\text{Tire revolutions/mi}) (\text{MPH})}$$

What should the axle ratio be if the engine rpm is 1950, the loaded diameter of the tire is 36" and the desired speed is 50 mph. ($\pi = 3.14$)

10. Find the axle ratio if the engine rpm is 2400, the loaded diameter of the tire is 32 inches, and the desired road speed is 40 mph. ($\pi = 22/7$)

FORMULAS
Level 2

Diesel Mechanic

Complete Solutions to B Problems

4.
$$\text{RPM} = \frac{(168)(10)(15 \text{ mph})}{19.5 \text{ in}} = 1292 \text{ rpm (nearest unit)}$$

5.
$$\text{AR} = (0.0025)(40^2)(80) = 320 \text{ lbs}$$

6.
$$\text{HP} = \frac{(40^3)(80)}{150,000} = 34 \text{ hp (nearest unit)}$$

7.
$$\text{T} = (2)(0.25)(5000)(0.5281) \approx 1320 \text{ lbs-ft (nearest unit)}$$

8.
$$\text{bmep} = \frac{(210)(33,000)}{(18.66)(1/2)(5400)} = 137.55$$

9. The circumference of the tire is $C = \pi d = (3.14)(3 \text{ ft}) = 9.42 \text{ ft}$

Then, the tire revs per mile is $\frac{5280 \text{ ft}}{9.42 \text{ ft}} = 560.5$

Axle ratio = $\frac{(60)(1950)}{(560.5)(50)} = 4.17$

10. Tire rev/mi: $\frac{5280}{8/3\pi} \approx 630$

Axle Ratio = $\frac{(60)(2400)}{(630)(40)} \approx 5.71$

USE OF FORMULAS
Level 1

Auto Mechanics

A. Problems with Solutions

1. The amperage through a solenoid is 3.4 amps. If the battery is a 12 volt battery, what is the internal resistance of the solenoid?

Definitions: amps: amperes, V: volts, R: resistance, I: amperes

Solution:

$$R = \frac{V}{I}$$

$$R = \frac{12 \text{ volts}}{3.4 \text{ amps}} = 3.53 \text{ Ohms}$$

2. Indicated horsepower is considered to be pure power of an engine without loss of parasitic or frictional horsepower and is computed by:

$$HP = \frac{PLANK}{33,000}$$

and P = mean effective pressure

L = length of stroke in feet

A = area of piston

N = number of pistons

$$K = \frac{RPM}{2}$$

Solve for P.

Definitions: HP: horsepower, Parasitic horsepower: that horsepower required to operate equipment necessary to keep engine running i.e., oil pump, water pump. RPM: revolutions per minute

Solution:

$$P = \frac{(33,000) (HP)}{LANK}$$

3. The pitch diameter of the crankshaft pulley on a given engine is 4.96". What is the belt speed if the engine is turning 2,850 RPM?

Solution:

$$\begin{aligned} \text{Belt speed} &= \frac{\pi (4.96 \text{ in}) (2,850 \text{ RPM})}{12 \text{ in/ft}} \\ &= \frac{(3.1416) (4.96 \text{ in}) (2,850 \text{ RPM})}{12 \text{ in/ft}} = 3,700.8 \text{ ft/min} \end{aligned}$$

USE OF FORMULAS
Level 1

Auto Mechanics

B. Problems without Solutions

4. What are the revolutions per mile of an engine with a 4.1 : 1 rear end ratio and a 6.70 x 15 tire? (28" diam)

5. The oil pressure on a given engine should be 40 PSI. If the relief valve plunger is 1/2" diameter, how much force must the relief spring exert?

Definitions: PSI = pounds per square inch, $P \times A = F$ (pressure x area = force)

6. A tire on a 1972 Pinto (23" wheel diameter) has a 1 ounce heavy spot in the tread. At 60 miles per hour, what eccentric force would this unbalanced wheel exert? (This would be the same force with which the heavy spot would strike the road.)

Definitions: F: force in lbs or Newtons, m: mass in slugs or Kg

$m = \frac{\text{weight}}{\text{gravity}}$, V: velocity in ft/sec or meter/sec

r: radius in feet or meters

USE OF FORMULAS
Level 1

Auto Mechanics

B. Problems without Solutions (continued)

7. A vehicle with a $1 \frac{1}{16}$ inch solid steel drive shaft which is 5 ft 4 inches long has a maximum torque of 550 ft lbs. What maximum twist could be imposed on the drive shaft?

USE OF FORMULAS
Level 1

Auto Mechanics

Complete Solutions to B Problems

$$4. \quad \left[\frac{5,280 \text{ ft/mile}}{\left(\frac{28 \text{ in}}{12 \text{ in/ft}} \right) (\pi)} \right] (4.1 \text{ rev}) = \frac{(5,280 \text{ ft/mile}) (12 \text{ in/ft}) (4.1 \text{ rev})}{(28 \text{ in}) (3.1416)}$$

$$= 2,953.18 \text{ rev/mile}$$

$$5. \quad A = \pi r^2$$

$$A = \pi \left(\frac{d}{2} \right)^2$$

$$\text{Force} = PA = (40 \text{ lbs/sq in}) (3.1416) \left(\frac{1/2"}{2} \right)^2$$

$$= (40 \text{ lbs/sq in}) (3.1416) (1/4 \text{ sq in}) (1/4)$$

$$= 7.85 \text{ lb exerted by relief spring}$$

$$6. \quad F = m \frac{V}{r}$$

$$F = \left(\frac{1/16 \text{ lb}}{32 \text{ ft/sec}^2} \right) \left(\frac{(88 \text{ ft/sec})^2}{0.48 \text{ ft}} \right)$$

$$F = \frac{(1/16 \text{ lb}) (7,744 \text{ sq ft/sec}^2)}{(32 \text{ ft/sec}^2) (0.48 \text{ ft})}$$

$$F = 31.51 \text{ lbs eccentric force}$$

$$7. \quad A = \frac{(584) TL}{D^4 G}$$

$$A = \frac{(584) (550 \text{ ft lbs}) (12 \text{ in/ft}) (64 \text{ in})}{(17/16)^4 (11,500,000)}$$

$$A = 16.8^\circ$$

FORMULAS

$$\text{Circumference} = \pi \times \text{dia}$$

$$\text{Area} = \pi \times R \text{ sq or } \frac{\pi \times \text{dia}^2}{4} \text{ sq or } \text{dia}^2 \times 0.785$$

$$\text{Volume of cylinder} = \text{surface} \times \text{depth}$$

$$\text{Volume of cone} = \text{surface area} \times \frac{\text{height}}{3}$$

$$\text{Volume of pyramid} = \frac{\text{base area} \times \text{height}}{3}$$

$$\text{Area of triangle} = \frac{\text{base} \times \text{height}}{2}$$

$$\text{Area of rectangle} = l \times w$$

$$\text{Volume of rectangle} = l \times w \times h$$

FLOW MEASUREMENTS

$$Q = \text{rate of flow}$$

$$V = \text{total volume of flow} \quad V = \text{velocity in ft/sec}$$

$$T = \text{time}$$

$$A = \text{area}$$

$$Q = \frac{\text{pumping time} \times \text{rate}}{\text{time period}}$$

$$T = \frac{V}{\text{flow in time}}$$

$$Q \text{ in cfs} = V \times A$$

$$Q \text{ in gpm} = V \times A \times 450$$

$$v = \frac{Q}{A}$$

$$W = \text{pounds B.O.C. applied}$$

$$A = \text{area of filter in acres}$$

$$\text{Filter Eff} = \frac{100}{1 + (0.0025) \left(\frac{W}{A \times D \times P} \right)}$$

$$Q = \text{GPM} \times \frac{\text{new}}{\text{old}} = \text{new gpm}$$

$$H = \text{feet} \times \frac{(\text{new})^2}{\text{old}} = \text{new feet}$$

$$P = \text{hp} \times \frac{(\text{new})^3}{\text{old}} = \text{new hp}$$

$$D = \text{depth of filter media}$$

$$F = \text{recirculation ratio}$$

FORMULAS

$$D.T. = \frac{\text{vol in gals} \times 24}{\text{flow /day}}$$

$$\text{Weir loading} = \frac{\text{flow}}{\text{weir length}} = \text{gal/ft of weir/day}$$

$$\text{Overflow rate} = \frac{\text{flow}}{\text{surface area}} = \text{gal/sq ft/day}$$

$$\text{Area of filter (acres)} = \frac{\text{dia squared} \times 0.785}{43560} = \text{Acres}$$

$$\text{Hydraulic loading} = \frac{\text{MGD}}{\text{acres}} = \text{MG/acre/day}$$

$$\text{Recirculation ratio} = \frac{\text{return}}{\text{flow}}$$

$$\% \text{ Removal} = \frac{\text{influent-effluent}}{\text{influent}} \times 100$$

$$\text{Volume} = \text{dia sq} \times 0.785 \times 7.5 \times \text{depth} = \text{vol in gals}$$

$$\text{Pounds applied} = \text{flow} \times 8.34 \times \text{PPM} = \text{pounds/day}$$

$$\text{B.O.D. Loading} = \frac{\text{pounds/day}}{\text{volume}} \times 1000 = \text{lbs of B.O.D. cu ft of medium/day}$$

$$W_1 \times L_1 = W_2 \times L_2 \quad (\text{Area being equal})$$

$$1 \text{ cu ft} = 1,728 \text{ cu in}$$

$$7.5 \text{ gal} = 1 \text{ cu ft}$$

$$1 \text{ cu ft water} = 62.4 \text{ pds}$$

$$1 \text{ gal} = 8.34 \text{ lbs}$$

$$1 \text{ cfs} = 450 \text{ GPM}$$

$$\pi = 3.14$$

$$1 \text{ cfs} = 646,317 \text{ gal/day}$$

$$1/4 \pi = 0.785$$

$$W = \text{width}$$

$$L = \text{length}$$

$$H = \text{height}$$

$$\text{B.O.D.} = \text{initial D.O.} - 5 \text{ day D.O.} \times \frac{300}{\text{ml of sample}}$$

$$P_1 \times V_1 = P_2 \times V_2 \quad (\text{If temperature is the same})$$